9-5 Functions and Their Inverses

Objectives

Determine whether the inverse of a function is a function.

Write rules for the inverses of functions.

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Vocabulary

one-to-one function

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In Lesson 7-2, you learned that the inverse of a function $f(x)$ "undoes" $f(x)$. Its graph is a reflection across line $y = x$. The inverse may or not be a function.

Recall that the vertical-line test (Lesson 1-6) can help you determine whether a relation is a function. Similarly, the *horizontal-line* test can help you determine whether the inverse of a function is a function.

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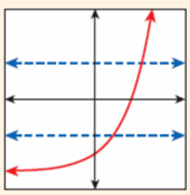
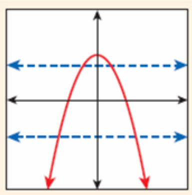
Horizontal-line Test

WORDS	EXAMPLES	
If any horizontal line passes through more than one point on the graph of a relation, the inverse relation is not a function.	 Inverse is a function.	 Inverse is not a function.

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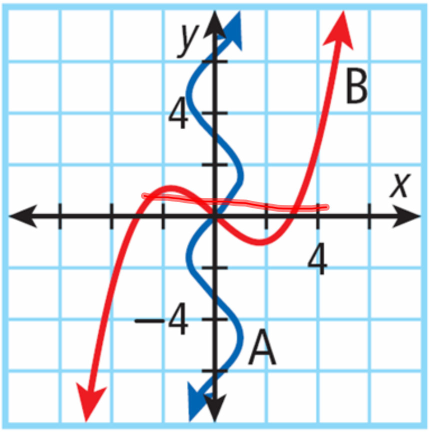
Example 1A: Using the Horizontal-Line Test

Use the horizontal-line test to determine whether the inverse of the blue relation is a function.



Inverse is function

Inverse is not a function

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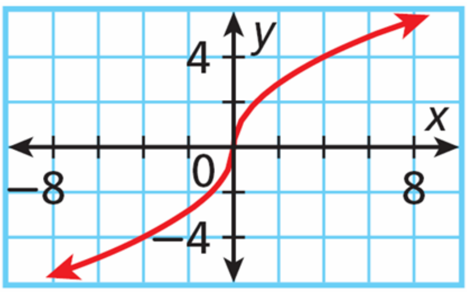
Check It Out! Example 1

Use the horizontal-line test to determine whether the inverse of each relation is a function.



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Recall from Lesson 7-2 that to write the rule for the inverse of a function, you can exchange *x* and *y* and solve the equation for *y*. Because the value of *x* and *y* are switched, the domain of the function will be the range of its inverse and vice versa.

Find the inverse of $f(x) = \sqrt[3]{x+1}$. Determine whether it is a function, and state its domain and range.

$y = \sqrt[3]{x+1}$

$(x)^3 = \left(\sqrt[3]{y+1}\right)^3$

$x^3 = y+1$

$x^3 - 1 = y$

D: $\mathbb{R}$

R: $\mathbb{R}$

Find the inverse of $f(x) = x^3 - 2$. Determine whether it is a function, and state its domain and range.

$y = x^3 - 2$

$x = y^3 - 2$

$x + 2 = y^3$

$\sqrt[3]{x+2} = y$

You have seen that the inverses of functions are not necessarily functions. When both a relation and its inverses are functions, the relation is called a *one-to-one function*. In a **one-to-one function**, each *y*-value is paired with exactly one *x*-value.

You can use composition of functions to verify that two functions are inverses. Because inverse functions "undo" each other, when you compose two inverses the result is the input value *x*.

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Identifying Inverse Functions

WORDS	ALGEBRA	EXAMPLE
If the compositions of two functions equal the input value, the functions are inverses.	If $f(g(x)) = g(f(x)) = x$, then $f(x)$ and $g(x)$ are inverse functions.	$f(x) = 3x$ and $g(x) = \frac{1}{3}x$ $f(g(x)) = 3\left(\frac{1}{3}x\right) = x$ $g(f(x)) = \frac{1}{3}(3x) = x$

Determine by composition whether each pair of functions are inverses.

$f(x) = 3x - 1$ and $g(x) = \frac{1}{3}x + 1$

$3\left(\frac{1}{3}x+1\right) - 1$

$x + 3 - 1$

$x + 2$

x

$\frac{1}{3}(3x-1) + 1$

$x - \frac{1}{3} + 1$

$x + \frac{2}{3}$

x

For $x \neq 1$ or 0, $f(x) = \dfrac{1}{x-1}$ and $g(x) = \dfrac{1}{x} + 1$.

$$\frac{1}{\left(\frac{1}{x}+1\right)-1}$$

$$\frac{1}{\frac{1}{x}} \quad \frac{1}{1} \cdot \frac{x}{1}$$

$$x$$

$$\frac{1}{x-1} + 1$$

$$x-1 + 1(x-1)$$

$$x-1+x-1$$

$$2x-2$$

Determine by composition whether each pair of functions are inverses.

$f(x) = \dfrac{2}{3}x + 6$ and $g(x) = \dfrac{3}{2}x - 9$

$$\frac{2}{3}\left(\frac{3}{2}x-9\right)+6$$

$$x-6+6$$

$$x$$

$$\frac{3}{2}\left(\frac{2}{3}x+6\right)-9$$

$$x+9-9$$

$$x$$

$f(x) = x^2 + 5$ and $g(x) = \sqrt{x} - 5$ for $x \geq 0$

$(\sqrt{x} - 5)^2 + 5$

$x - 10\sqrt{x} + 25 + 5$

$(\sqrt{x} - 5)(\sqrt{x} - 5)$

$x - 10\sqrt{x} + 30$

Homework:

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