Warm Up

Simplify. Assume that all expressions are defined.

1. $(2x + 5) - (x^2 + 3x - 2)$

$-x^2 - x + 7$

2. $(x - 3)(x + 1)^2$

$(x-3)(x^2+2x+1)$

3. $\dfrac{x^2 - x - 6}{x^2 - 4}$

$$\frac{(x-3)\cancel{(x+2)}}{(x-2)\cancel{(x+2)}}$$

8. $g(x) = \begin{cases} \frac{1}{4}x^2 & \text{if } x < 1 \\ x & \text{if } x \geq 1 \end{cases}$

9. $h(x) = \begin{cases} \left(\frac{x}{2}\right)^2 & \text{if } x < 2 \\ 2x & \text{if } x \geq 2 \end{cases}$

10. $p(x) = \begin{cases} x^2 - 3 & \text{if } x < 1 \\ 4x - 3 & \text{if } x \geq 1 \end{cases}$

11. $f(x)$: x-int. = 6, y-int. = 9;
$g(x)$: x-int. = 6, y-int. = 6

12. $f(x)$: x-int. = ± 5, y-int. = -25;
$g(x)$: x-int. = ± 3, y-int. = -25

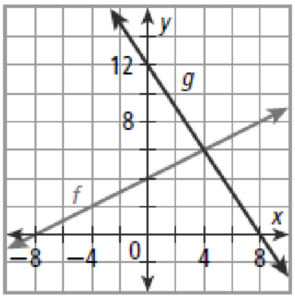
13. $f(x)$: x-int. = 5, y-int. = 2;
$g(x)$: x-int. = 2.5, y-int. = 2

14. $f(x)$: x-int. = -1 and 4, y-int. = -4;
$g(x)$: x-int. = -3 and 12, y-int. = 4

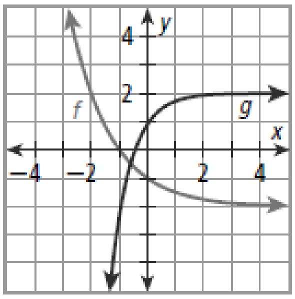
15. $f(x)$: x-int. $= 0$, y-int. $= 0$;
$g(x)$: x-int. $= 1$, y-int. $= -4$

16. $f(x)$: x-int. $= -2$, y-int. $= 8$;
$g(x)$: x-int. $= 4$, y-int. $= 8$

17.



18.



22a. $f(x) = \begin{cases} 6 & \text{if } x \leq 200 \\ 0.03x & \text{if } x > 200 \end{cases}$

b. $f(x) = \begin{cases} 6.90 & \text{if } 0 < x \leq 200 \\ 0.0345x & \text{if } x > 200 \end{cases}$

25. $f(x) - 7 = \begin{cases} 2^x - 8 & \text{if } x \leq -3 \\ -5x - 4 & \text{if } x > -3 \end{cases}$

26. $5f(x) = \begin{cases} 15x^2 & \text{if } x < 1 \\ 5(-2x + 4) & \text{if } x \geq 1 \end{cases}$

32. C

33. J;

$$g(x) = \begin{cases} 2(4x) & \text{if } 4x > 8 \\ (4x)^2 & \text{if } 4x \leq 8 \end{cases}$$

$$= \begin{cases} 8x & \text{if } 4x > 8 \\ 16x^2 & \text{if } 4x \leq 8 \end{cases}$$

34. C;

$$f(5x) = \frac{5}{3}g(x)$$

$$15 \times \frac{5}{3} = 25$$

9-4 Operations with Functions

Objectives

Add, subtract, multiply, and divide functions.

Write and evaluate composite functions.

9-4 Operations with Functions

Vocabulary

composition of functions

9-4 Operations with Functions

You can perform operations on functions in much the same way that you perform operations on numbers or expressions. You can add, subtract, multiply, or divide functions by operating on their rules.

9-4 Operations with Functions

Notation for Function Operations	
Operation	**Notation**
Addition	$(f+g)(x) = f(x) + g(x)$
Subtraction	$(f-g)(x) = f(x) - g(x)$
Multiplication	$(fg)(x) = f(x) \cdot g(x)$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, where $g(x) \neq 0$

Holt Algebra 2
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Do the operation the problem says and simplify as much as possible. It will make life a lot easier if you have to evaluate for a certain point on the function.

Given $f(x) = 4x^2 + 3x - 1$ and $g(x) = 6x + 2$, find each function.

$(f + g)(x)$

$f(x) + g(x)$

$4x^2 + 3x - 1 + 6x + 2$

$(f+g)(x) = 4x^2 + 9x + 1$

Given $f(x) = 4x^2 + 3x - 1$ and $g(x) = 6x + 2$, find each function.

$(f - g)(x)$

$4x^2+3x-1-(6x+2)$

$4x^2-3x-3$

Given $f(x) = 5x - 6$ and $g(x) = x^2 - 5x + 6$, find each function.

$(f + g)(x)$

$5x-6+x^2-5x+6$

x^2

Given $f(x) = 5x - 6$ and $g(x) = x^2 - 5x + 6$, find each function.

$(f - g)(x)$

When you divide functions, be sure to note any domain restrictions that may arise.

Also remember when multiplying you must us FOIL, don't just do first times first and second times second!

Given $f(x) = 6x^2 - x - 12$ and $g(x) = 2x - 3$, find each function.

$(fg)(x)$

$(6y^2 - x - 12)(2x - 3)$

$12x^3 - 18x^2 - 2x^2 + 3x - 24x + 36$

$12x^3 - 20x^2 - 21x + 36$

$\left(\frac{f}{g}\right)(x)$

Given $f(x) = x + 2$ and $g(x) = x^2 - 4$, find each function.

$(fg)(x)$

$(x+2)(x^2-4)$

$x^3-4x+2x^2-8$

x^3+2x^2-4x-8

$\left(\frac{g}{f}\right)(x)$ $f(x)=x+2$ $g(x)=x^2-4$

$\frac{x^2-4}{x+2}$ $x \neq -2$

$\frac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}}$

$x-2$

9-4 Operations with Functions

Another function operation uses the output from one function as the input for a second function. This operation is called the **composition of functions**.

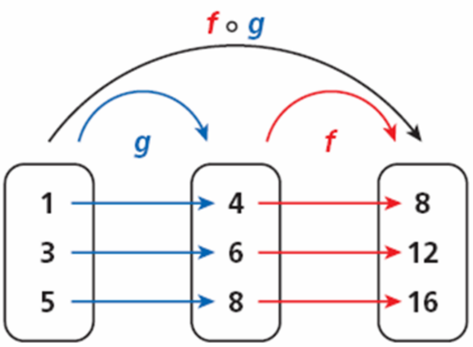
Composition of Functions

The composition of functions f and g is notated

$$(f \circ g)(x) = f(g(x)).$$

The domain of $(f \circ g)(x)$ is all values of x in the domain of g such that $g(x)$ is in the domain of f.

9-4 Operations with Functions



To find $(f \circ g)(1)$, first find $g(1)$.

$$g(1) = 4$$

Then use 4 as the input into f:

$$f(4) = 8$$

So $(f \circ g)(1) = f(g(1)) = 8$.

The order of function operations is the same as the order of operations for numbers and expressions. To find $f(g(3))$, evaluate $g(3)$ first and then substitute the result into f.

9-4 Operations with Functions

Reading Math

The composition $(f \circ g)(x)$ or $f(g(x))$ is read "f of g of x."

For these problems:
1)Plug in the x value on the inner most function
2)Solve
3)Take the value you just found and plug into the outer function
4)Solve for your answer

Given $f(x) = 2^x$ and $g(x) = 7 - x$, find each value.

$f(g(4))$

$g(4) = 7 - 4 = 3$

$f(g(4)) = 2^3 = 8$

Given $f(x) = 2^x$ and $g(x) = 7 - x$, find each value.

$g(f(4))$

$f(4) = 2^4 = 16$

$g(f(4)) = 7 - 16 = -9$

Given $f(x) = 2x - 3$ and $g(x) = x^2$, find each value.

$f(g(3))$

$g(3) = 3^2 = 9$

$f(g(3)) = 2(9) - 3 = 15$

Given $f(x) = 2x - 3$ and $g(x) = x^2$, find each value.

$g(f(3))$

9-4 Operations with Functions

You can use algebraic expressions as well as numbers as inputs into functions. To find a rule for $f(g(x))$, substitute the rule for g into f.

Given $f(x) = x^2 - 1$ and $g(x) = \frac{x}{1-x}$, write each composite function. State the domain of each.

$f(g(x))$

$$f(g(x)) = \left(\frac{x}{1-x}\right)^2 - 1$$

$$f(g(x)) = \frac{x^2}{(1-x)^2} - 1$$

Given $f(x) = x^2 - 1$ and $g(x) = \frac{x}{1-x}$, write each composite function. State the domain of each.

$g(f(x))$

$$\frac{x^2-1}{1-(x^2-1)} = \frac{x^2-1}{-x^2}$$

D: $x \neq 0$, $\mathbb{R}$

Given $f(x) = 3x - 4$ and $g(x) = \sqrt{x} + 2$, write each composite. State the domain of each.

$f(g(x))$

$3(\sqrt{x} + 2) - 4$

$3\sqrt{x} + 6 - 4$

$3\sqrt{x} + 2$

D: $x \geq 0$

Given $f(x) = 3x - 4$ and $g(x) = \sqrt{x} + 2$, write each composite. State the domain of each.

$g(f(x))$

$3x-4=0$
$3x=4$
$x=\frac{4}{3}$

$\sqrt{3x-4}+2$

D: $x \geq \frac{4}{3}$

Make up two functions. Find:

(f-g)(x)

(f+g)(x)

(f/g)(x)

(fg)(x)

During a sale, a music store is selling all drum kits for 20% off. Preferred customers also receive an additional 15% off.

$f(x)$ $g(x)$

a. Write a composite function to represent the final cost of a kit for a preferred customer that originally cost *c* dollars.

$f(x) = .8c$

$g(x) = .85c$

$g(f(x))$

$f(g(x))$

$.8(.85c)$

$.68c$

$.85(.8c)$

$.68c$

32% off

Homework:

p. 686 #15-32, 45-47, 50