

Warm Up

Simplify each expression. Assume all variables are positive.

$$1. \frac{2\sqrt{27x} + 3\sqrt{12x}}{2\sqrt{9}\sqrt{3x} + 3\sqrt{4}\sqrt{3x}}$$

$$2. \sqrt{72y^5}$$

$$3. \sqrt[3]{(x+2)^3} \quad \frac{6\sqrt{3x} + 6\sqrt{3x}}{2\sqrt{3x}}$$

$$4. \sqrt{2(48y)}$$

8-8 Solving Radical Equations and Inequalities

Objective

Solve radical equations and inequalities.

8-8**Solving Radical Equations
and Inequalities*****Vocabulary***

radical equation
radical inequality

8-8**Solving Radical Equations
and Inequalities**

A **radical equation** contains a variable within a radical. Recall that you can solve quadratic equations by taking the square root of both sides. Similarly, radical equations can be solved by raising both sides to a power.

Solving Radical Equations	
Steps	Example
1. Isolate the radical.	$\sqrt[3]{x} - 2 = 0$ $\sqrt[3]{x} = 2$
2. Raise both sides of the equation to the power equal to the index of the radical.	$(\sqrt[3]{x})^3 = (2)^3$
3. Simplify and solve.	$x = 8$

Solve each equation.

$$5 + \sqrt{x + 1} = 16$$

$$\begin{array}{ccc} -5 & & -5 \\ \hline \sqrt{x+1} & = & 11 \end{array}$$

$$(\sqrt{x+1})^2 = (11)^2$$

$$\begin{array}{ccc} x+1 & = & 121 \\ -1 & -1 & \\ \hline x & = & 120 \end{array}$$

Solve each equation.

$$\frac{7\sqrt[3]{5x-7}}{7} = \frac{84}{7}$$

$$(\sqrt[3]{5x-7})^3 = (12)^3$$

$$\begin{array}{r} 5x-7 = 1728 \\ +7 \quad +7 \end{array}$$

$$\frac{5x}{5} = \frac{1735}{5}$$

$$x = 347$$

Solve the equation.

$$4 + \sqrt{x-1} = 5$$

Solve the equation.

$$\left(\sqrt[3]{3x-4}\right)^3 = 2^3$$

$$3x-4=8$$

$$3x=12$$

$$x=4$$

Solve $\left(\sqrt{7x+2}\right)^2 = \left(3\sqrt{3x-2}\right)^2$

$$7x+2=9(3x-2)$$

$$7x+2=27x-18$$

$$2=20x-18$$

$$20=20x$$

$$x=1$$

Solve each equation.

$$\sqrt{8x+6}^2 = 3\sqrt{x}^2$$

$$\begin{aligned}8x+6 &= 9x \\ -8x & \quad -8x \\ 6 &= x\end{aligned}$$

Raising each side of an equation to an even power may introduce extraneous solutions.

Helpful Hint

You can use the intersect feature on a graphing calculator to find the point where the two curves intersect.

Because there is the possibility of an extraneous solution you **have to** check every solution you find.

Solve $\sqrt{-3x + 33} = (5 - x)(5 - x)$

$$-3x + 33 = 25 - 10x + x^2$$

$$-8 - 7x + x^2$$

Solve each equation.

$$\sqrt{2x+14} = x+3$$

$$2x+14 = x^2+6x+9$$

$$x^2+4x-5=0$$

$$(x+5)(x-1)=0$$

$$x = -5 \text{ or } 1$$

$$4=4 \checkmark$$

$$\sqrt{2 \cdot -5 + 14} = -2$$

$$2 = -2x$$

Solve each equation.

$$\sqrt{-9x+28} = -x+4$$

Remember!

To find a power, multiply the exponents.

$$\left[(x + 12)^{\frac{1}{2}} \right]^2$$

$$(x + 12)^{\frac{1}{2} \cdot 2}$$

$$x + 12$$

$$\frac{1}{2}$$

$$\sqrt{\quad}$$

$$\frac{3}{4} \cdot \frac{4}{3}$$

Solve each equation.

$$\left((5x + 7)^{\frac{1}{3}} \right)^3 = 3^3$$

$$5x + 7 = 27$$

$$\quad -7 \quad -7$$

$$5x = 20$$

$$x = 4$$

Solve each equation.

$$\left((x + 5)^{\frac{1}{3}} \right)^3 = 3^3$$

$$x + 5 = 27$$

$$x = 22$$

A **radical inequality** is an inequality that contains a variable within a radical. You can solve radical inequalities by graphing or using algebra.

Remember!

A radical expression with an even index and a negative radicand has no real roots.

When you solve these problems you need to also "check" your answer. Is your answer allowed? This question should ring through your mind.

This happens because say you get an x that makes a square root negative. This isn't allowed so you may need to adjust your solution accordingly.

Solve $\sqrt{2x-6} + 3 \leq 9$.

$$\begin{array}{l} 2x-6=6 \\ 2x=6 \\ x=3 \end{array}$$

$$\sqrt{2x-6} \leq 6$$

$$\begin{array}{l} 2x-6 \leq 36 \\ +6 \quad +6 \end{array}$$

$$\frac{2x}{2} \leq \frac{42}{2}$$

$$3 \leq x \leq 21$$

$\sqrt{36}$

Solve $\sqrt{x-3} + 2 \leq 5.$

$$\sqrt{x-3} \leq 3$$

$$x-3 \leq 9$$

$$3 \leq x \leq 12$$

$$x-3=0$$

$$x=3$$

Solve $\sqrt[3]{x+2} \geq 1.$

$$x+2 \geq 1$$

$$x \geq -1$$

$$\sqrt[3]{-1+2} = \sqrt[3]{1} = 1$$

Homework:

p. 633 #28-44(even), 47-49, 54-57, 71-73