Warm Up:

Simplify each expression.

1. $7^3 \bullet 7^2 = 7^5$

2. $\frac{11^8}{11^6} = 11^2$

3. $(3^2)^3 = 3^6$

4. $\sqrt{75} = \sqrt{25 \cdot 3} = \sqrt{25} \cdot \sqrt{3}$

$5\sqrt{3}$

5. $\frac{\sqrt{20}}{\sqrt{7}} = \frac{\sqrt{5 \cdot 4}}{\sqrt{7}} = \frac{2\sqrt{5}}{\sqrt{7}}$

19. $4 + \frac{1}{x} = \frac{10}{2x}$

$$4(2x) + \frac{1}{x}(2x) = \frac{10}{2x}(2x)$$
$$8x + 2 = 10$$
$$8x = 8$$

20. $\frac{5}{4} = \frac{n-3}{n-4}$

$$5(n-4) = 4(n-3)$$
$$5n - 20 = 4n - 12$$
$$n = 8$$

21. $\frac{1}{a-7} = 3$

$$1 = 3(a-7)$$
$$\frac{1}{3} = a - 7$$
$$\frac{22}{3} = a$$

22. $\frac{1}{x} - \frac{3}{4} = \frac{x}{4}$

$$\frac{1}{x}(4x) - \frac{3}{4}(4x) = \frac{x}{4}(4x)$$
$$4 - 3x = x^2$$
$$x^2 + 3x - 4 = 0$$
$$(x-1)(x+4) = 0$$
$$x = 1 \text{ or } x = -4$$

23. $\frac{14}{z} = 9 - z$

$$\frac{14}{z}(z) = 9(z) - z(z)$$
$$14 = 9z - z^2$$
$$z^2 - 9z + 14 = 0$$
$$(z-2)(z-7) = 0$$
$$z = 2 \text{ or } z = 7$$

24. $x + \frac{4}{x} = 4$

$$x(x) + \frac{4}{x}(x) = 4(x)$$
$$x^2 + 4 = 4x$$
$$x^2 - 4x + 4 = 0$$
$$(x-2)^2 = 0$$
$$x - 2 = 0$$
$$x = 2$$

25. $\frac{4x}{x-3} + \frac{x}{2} = \frac{12}{x-3}$

$$\left(\frac{4x}{x-3}\right)2(x-3) + \left(\frac{x}{2}\right)2(x-3) = \left(\frac{12}{x-3}\right)2(x-3)$$
$$8x + x(x-3) = 24$$
$$x^2 + 5x - 24 = 0$$
$$(x-3)(x+8) = 0$$
$$x = 3 \text{ or } x = -8$$

The solution $x = 3$ is extraneous.
The only solution is $x = -8$.

26. $\frac{3x}{x+1} = \frac{2x-1}{x+1}$

$$\frac{3x}{x+1}(x+1) = \frac{2x-1}{x+1}(x+1)$$
$$3x = 2x - 1$$
$$x = -1$$

The solution $x = -1$ is extraneous.
Therefore there is no solution.

27. $\frac{2}{x(x-1)} = 1 + \frac{2}{x-1}$

$$\left(\frac{2}{x(x-1)}\right)x(x-1) = 1(x(x-1)) + \left(\frac{2}{x-1}\right)x(x-1)$$
$$2 = x(x-1) + 2x$$
$$x^2 + x - 2 = 0$$
$$(x-1)(x+2) = 0$$
$$x = 1 \text{ or } x = -2$$

The solution $x = 1$ is extraneous.
The only solution is $x = -2$.

50. $\frac{1}{x} + \frac{7}{2} = 2$

$$\frac{1}{x} = -\frac{3}{2}$$
$$x = -\frac{2}{3}$$

8-6 Radical Expressions and Rational Exponents

Objectives

Rewrite radical expressions by using rational exponents.

Simplify and evaluate radical expressions and expressions containing rational exponents.

8-6 Radical Expressions and Rational Exponents

Vocabulary

index

rational exponent

8-6 Radical Expressions and Rational Exponents

You are probably familiar with finding the square root of a number. These two operations are inverses of each other. Similarly, there are roots that correspond to larger powers.

5 and –5 are square roots of 25 because $5^2 = 25$ and $(-5)^2 = 25$

2 is the cube root of 8 because $2^3 = 8$.

2 and –2 are fourth roots of 16 because $2^4 = 16$ and $(-2)^4 = 16$.

a is the nth root of b if $a^n = b$.

8-6 Radical Expressions and Rational Exponents

The *n*th root of a real number *a* can be written as the radical expression $\sqrt[n]{a}$, where *n* is the **index** (plural: *indices*) of the radical and *a* is the radicand. When a number has more than one root, the radical sign indicates only the principal, or positive, root.

Numbers and Types of Real Roots

Case	Roots	Example
Odd index	1 real root	The real 3rd root of 8 is 2.
Even index; positive radicand	2 real roots	The real 4th roots of 16 are ±2.
Even index; negative radicand	0 real roots	–16 has no real 4th roots.
Radicand of 0	1 root of 0	The 3rd root of 0 is 0.

When you have an even index you need to find both the positive and negative roots.

When you have an odd index just find the root with the same sign as the radical number.

Find all real roots.

2^6

A. sixth roots of 64

$\sqrt[6]{64} = \pm 2$

B. cube roots of –216

$\sqrt[3]{-216} = -6$

C. fourth roots of –1024

$\sqrt[4]{-1024}$: DNE

Find all real roots.

a. fourth roots of −256

$\sqrt[4]{-256} =$ DNE

b. sixth roots of 1

$\sqrt[6]{1} = 1, -1$

c. cube roots of 125

$\sqrt[3]{125} = 5$

The properties of square roots in Lesson 1-3 also apply to *n*th roots.

Properties of *n*th Roots

For $a > 0$ and $b > 0$,

WORDS	NUMBERS	ALGEBRA
Product Property of Roots The *n*th root of a product is equal to the product of the *n*th roots.	$\sqrt[3]{16} = \sqrt[3]{8} \cdot \sqrt[3]{2} = 2\sqrt[3]{2}$	$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
Quotient Property of Roots The *n*th root of a quotient is equal to the quotient of the *n*th roots.	$\sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4}$	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

To simplify expressions just break into multiple roots and try to cancel things out by simplifying. Your answer should look "nicer" than when you started.

Simplify each expression. Assume that all variables are positive.

$$\sqrt[4]{81x^{12}}$$

$$\sqrt[4]{81} \cdot \sqrt[4]{x^{12}}$$

$$3x^3$$

Simplify the expression. Assume that all variables are positive.

$$\sqrt[4]{16x^4}$$

$$\sqrt[4]{16} \cdot \sqrt[4]{x^4}$$

$$2x$$

Simplify the expression. Assume that all variables are positive.

$$\sqrt[4]{\frac{x^8}{3}}$$

$$\frac{\sqrt[4]{x^8}}{\sqrt[4]{3}} = \frac{x^2}{\sqrt[4]{3}} \cdot \frac{\sqrt[4]{3}}{\sqrt[4]{3}} = \frac{x^2\sqrt[4]{3}}{3}$$

A **rational exponent** is an exponent that can be expressed as $\frac{m}{n}$, where m and n are integers and $n \neq 0$. Radical expressions can be written by using rational exponents.

Rational Exponents

For any natural number n and integer m,

WORDS	NUMBERS	ALGEBRA
The exponent $\frac{1}{n}$ indicates the nth root.	$16^{\frac{1}{4}} = \sqrt[4]{16} = 2$	$a^{\frac{1}{n}} = \sqrt[n]{a}$
The exponent $\frac{m}{n}$ indicates the nth root raised to the mth power.	$8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$	$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

8-6 Radical Expressions and Rational Exponents

Writing Math

The denominator of a rational exponent becomes the index of the radical.

To simplify a rational exponent first simplify the root and then raise to the power.

Write the expression $(-32)^{\frac{3}{5}}$ in radical form and simplify.

$$\left(\sqrt[5]{-32}\right)^3$$

$$(-2)^3$$

$$= -2 \cdot -2 \cdot -2$$

$$= -8$$

Write the expression $64^{\frac{1}{3}}$ in radical form, and simplify.

$$\left(\sqrt[3]{64}\right)^1$$

$$(4)^1$$

$$=4$$

Write the expression $4^{\frac{5}{2}}$ in radical form, and simplify.

$$\left(\sqrt[2]{4}\right)^5$$

$$(2)^5$$

$$32$$

Write the expression $625^{\frac{3}{4}}$ in radical form, and simplify.

$\left(\sqrt[4]{625}\right)^3$

$(5)^3$

125

Write each expression by using rational exponents.

A. $\sqrt[8]{13^4}$

$13^{\frac{4}{8}}$

3.61

B. $\sqrt[5]{13^{15}}$

$13^{\frac{15}{5}}$

Write each expression by using rational exponents.

a. $\left(\sqrt[4]{81}\right)^3$

$81^{\frac{3}{4}}$

b. $\sqrt[3]{10^9}$

$10^{\frac{9}{3}}$

c. $\sqrt[4]{5^2}$

$5^{\frac{2}{4}}$

8-6 Radical Expressions and Rational Exponents

Rational exponents have the same properties as integer exponents (See Lesson 1-5)

Properties of Rational Exponents

For all nonzero real numbers a and b and rational numbers m and n,

WORDS	NUMBERS	ALGEBRA
Product of Powers Property To multiply powers with the same base, add the exponents.	$12^{\frac{1}{2}} \cdot 12^{\frac{3}{2}} = 12^{\frac{1}{2}+\frac{3}{2}} = 12^2 = 144$	$a^m \cdot a^n = a^{m+n}$
Quotient of Powers Property To divide powers with the same base, subtract the exponents.	$\frac{125^{\frac{2}{3}}}{125^{\frac{1}{3}}} = 125^{\frac{2}{3}-\frac{1}{3}} = 125^{\frac{1}{3}} = 5$	$\frac{a^m}{a^n} = a^{m-n}$
Power of a Power Property To raise one power to another, multiply the exponents.	$\left(8^{\frac{2}{3}}\right)^3 = 8^{\frac{2}{3}\cdot 3} = 8^2 = 64$	$(a^m)^n = a^{m \cdot n}$
Power of a Product Property To find the power of a product, distribute the exponent.	$(16 \cdot 25)^{\frac{1}{2}} = 16^{\frac{1}{2}} \cdot 25^{\frac{1}{2}} = 4 \cdot 5 = 20$	$(ab)^m = a^m b^m$
Power of a Quotient Property To find the power of a quotient, distribute the exponent.	$\left(\frac{16}{81}\right)^{\frac{1}{4}} = \frac{16^{\frac{1}{4}}}{81^{\frac{1}{4}}} = \frac{2}{3}$	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

The properties remain the same, the only difference is now you will be working with fractions. And all fraction rules apply (need a common denom. to add, etc.)

Simplify each expression.

$7^{\frac{7}{9}} \bullet 7^{\frac{11}{9}}$

$7^{\frac{18}{9}} = 7^2 = 49$

Simplify each expression.

$$\frac{16^{\frac{3}{4}}}{16^{\frac{5}{4}}}$$

$$16^{\frac{-2}{4}} = 16^{-1/2}$$

$$= \frac{1}{16^{1/2}} = \frac{1}{4}$$

Simplify each expression.

$$36^{\frac{3}{8}} \bullet 36^{\frac{1}{8}}$$

$$36^{\frac{4}{8}} = 36^{\frac{1}{2}} = 6$$

Simplify each expression.

$$(-8)^{-\frac{1}{3}}$$

Simplify each expression.

$$\frac{5^{\frac{9}{4}}}{5^{\frac{1}{4}}}$$

Homework:

p. 615 #30-56(even), 59, 62-65, 73-76, 81