Warm up:

Simplify each expression. Assume all variables are nonzero.

1. $x^5 \bullet x^2$ x^7

2. $y^3 \bullet y^3$ y^6

3. $\frac{x^6}{x^2} = x^4$

4. $\frac{y^2}{y^5}$ $\frac{1}{y^3}$

Factor each expression.

5. $x^2 - 2x - 8$ $(x-4)(x+2)$

6. $x^2 - 5x$ $x(x-5)$

7. $x^5 - 9x^3$ $x^3(x^2-9) = x^3(x-3)(x+3)$

8-2 Multiplying and Dividing Rational Expressions

Objectives

Simplify rational expressions.

Multiply and divide rational expressions.

8-2 Multiplying and Dividing Rational Expressions

Vocabulary

rational expression

8-2 Multiplying and Dividing Rational Expressions

In Lesson 8-1, you worked with inverse variation functions such as $y = \frac{5}{x}$. The expression on the right side of this equation is a *rational expression.* A **rational expression** is a quotient of two polynomials. Other examples of rational expressions include the following:

$$\frac{x^2-4}{x+2} \qquad \frac{10}{x^2-6} \qquad \frac{x+3}{x-7}$$

8-2 Multiplying and Dividing Rational Expressions

Because rational expressions are ratios of polynomials, you can simplify them the same way as you simplify fractions. Recall that to write a fraction in simplest form, you can divide out common factors in the numerator and denominator.

$$\frac{9}{24} = \frac{3 \cdot \cancel{3}}{8 \cdot \cancel{3}} = \frac{3}{8}$$

Caution!

When identifying values for which a rational expression is undefined, identify the values of the variable that make the original denominator equal to 0.

Holt Algebra 2

What we are doing is simplifying algebraic expressions by canceling out terms from the top and bottom of a fraction.
To do this:

1) Identify which x-values you cannot have aka what makes the denominator 0. This can't happen because you cannot divide by 0.
2) Factor if possible
3) Cancel things out.

Simplify. Identify any x-values for which the expression is undefined.

$$\frac{10x^8}{6x^4}$$

$$6x^4 = 0$$

$$\frac{6x^4}{6} = \frac{0}{6}$$

$$\sqrt[4]{x^4} = \sqrt[4]{0}$$

$$x = 0$$

$$\frac{\overset{5}{\cancel{10}}\,x^{\cancel{8}\,4}}{\underset{3}{\cancel{6}}\,\cancel{x^4}} = \frac{5x^4}{3}$$

Simplify. Identify any x-values for which the expression is undefined.

$$\frac{x^2 + x - 2}{x^2 + 2x - 3}$$

$$\frac{(x+2)(x-1)}{(x+3)(x-1)}$$

$$(x+3)(x-1) = 0$$

$$x = -3 \text{ or } 1$$

$$\frac{(x+2)\cancel{(x-1)}}{(x+3)\cancel{(x-1)}} = \frac{x+2}{x+3}$$

Simplify. Identify any *x*-values for which the expression is undefined.

$$\frac{16x^{11}}{8x^2}$$

$$\frac{{}^{2}\cancel{16}x^{\cancel{11}\,9}}{{}_{1}\cancel{8}\cancel{x^2}} = 2x^9$$

$8x^2=0$

$x=0$

Simplify. Identify any *x*-values for which the expression is undefined.

$$\frac{3x+4}{3x^2+x-4} \rightarrow \frac{\cancel{3x+4}^{\,1}}{\cancel{(3x+4)}(x-1)} = \frac{1}{(x-1)}$$

$(3x+4)(x-1)=0$

$x=1$ or $\frac{-4}{3}$

Simplify. Identify any x-values for which the expression is undefined.

$$\frac{6x^2 + 7x + 2}{6x^2 - 5x - 5} \longrightarrow \frac{(2x+1)(3x+2)}{(3x+2)(2x-3)}$$

$(3x+2)(2x-3)=0$

$x = \frac{-2}{3}$ or $\frac{3}{2}$

$$\frac{2x+1}{2x-3}$$

Simplify $\frac{4x - x^2}{x^2 - 2x - 8}$. Identify any x values for which the expression is undefined.

$x^2-2x-8=0$

$(x-4)(x+2)=0$

$x=4$ or -2

$$\frac{4x-x^2}{(x-4)(x+2)} = \frac{-x(x-4)}{(x-4)(x+2)}$$

$$\frac{-x}{x+2}$$

Simplify $\frac{10 - 2x}{x - 5}$. Identify any x values for which the expression is undefined.

You can multiply rational expressions the same way that you multiply fractions.

Multiplying Rational Expressions
1. Factor all numerators and denominators completely.
2. Divide out common factors of the numerators and denominators.
3. Multiply numerators. Then multiply denominators.
4. Be sure the numerator and denominator have no common factors other than 1.

To multiply expressions:
1) Factor everything
2) Reduce what you can → cross reduce
3) Multiply top times top and bottom times bottom
4) Reduce

Multiply. Assume that all expressions are defined.

A. $\frac{3x^5y^3}{2x^3y^7} \cdot \frac{10x^3y^4}{9x^2y^5}$

$\frac{5x^3}{3y^5}$

B. $\frac{x-3}{4x+20} \cdot \frac{x+5}{x^2-9}$

$\frac{(x-3)}{4(x+5)} \cdot \frac{(x+5)}{(x-3)(x+3)}$

$\frac{1}{4(x+3)}$

Multiply. Assume that all expressions are defined.

A. $\frac{x}{15} \cdot \frac{x^7}{2x} \cdot \frac{20}{x^4}$

$\frac{x}{15} \cdot \frac{x^3}{x} \cdot \frac{10}{1}$

$\frac{2x^3}{3}$

B. $\frac{10x - 40}{x^2 - 6x + 8} \cdot \frac{x + 3}{5x + 15}$

$\frac{10(x-4)}{(x-4)(x+2)} \cdot \frac{(x+3)}{5(x+3)}$

$\frac{2}{x+2}$

To divide rational expressions:

1) Flip the second fraction and set up multiplication.
2) Factor everything
3) Cancel out what you can
4) Multiply and reduce

Divide. Assume that all expressions are defined.

$$\frac{5x^4}{8x^2y^2} \div \frac{15}{8y^5}$$

$$\frac{5x^{4\,2}}{8x^2y^2} \cdot \frac{8y^{5\,3}}{15_3}$$

$$\frac{x^2y^3}{3}$$

Divide. Assume that all expressions are defined.

$$\frac{x^4 - 9x^2}{x^2 - 4x + 3} \div \frac{x^4 + 2x^3 - 8x^2}{x^2 - 16}$$

$$\frac{(x^2)(x^2-9)}{(x-3)(x-1)} \div \frac{(x^2)(x^2+2x-8)}{(x-4)(x+4)}$$

$$\frac{x^2(x-3)(x+3)}{(x-3)(x-1)} \cdot \frac{(x-4)(x+4)}{x^2(x+4)(x-2)}$$

$$\frac{(x+3)(x-4)}{(x-1)(x-2)}$$

Divide. Assume that all expressions are defined.

$$\frac{x^2}{4} \div \frac{x^4 y}{12y^2}$$

To solve equations:

1) Factor everything
2) Cancel out what you can
3) Get x by itself

Solve. Check your solution.

$$\frac{x^2 - 25}{x - 5} = 14$$

$$\frac{(x-5)(x+5)}{(x-5)} = 14$$

$$x + 5 = 14$$
$$-5 \quad -5$$
$$x = 9$$

Solve. Check your solution.

$$\frac{x^2 - 3x - 10}{x - 2} = 7$$

$$-1 \cdot \frac{(x-5)(x+2)}{(x-2)} = 7 \cdot -1$$

$$\frac{(x+5)(x-2)}{(x-2)} = -7$$

$$x + 5 = -7$$

$$x = -12$$

Solve. Check your solution.

$$\frac{x^2 + x - 12}{x + 4} = -7$$

$$\frac{(x+4)(x-3)}{(x+4)} = -7$$

$$x - 3 = -7$$

$$x = -4$$

8-3 **Adding and Subtracting Rational Expressions**

Objectives

Add and subtract rational expressions.

Simplify complex fractions.

8-3 Adding and Subtracting Rational Expressions

Adding and subtracting rational expressions is similar to adding and subtracting fractions. To add or subtract rational expressions with like denominators, add or subtract the numerators and use the same denominator.

$$\frac{1}{5} + \frac{3}{5} = \frac{4}{5} \qquad \frac{6}{7} - \frac{4}{7} = \frac{2}{7}$$

Holt Algebra 2

To add or subtract fractions we need a common denominator. This is the same for rational expressions. For now they will give us one and we will discover how to find one later. But for now concentrate on the signs, either add or subtract. Remember if you subtract that minus sign applies to all of an expression not just the first term...

Add or subtract. Identify any *x*-values for which the expression is undefined.

$$\frac{x-3}{x+4} + \frac{x-2}{x+4}$$

Add or subtract. Identify any *x*-values for which the expression is undefined.

$$\frac{3x-4}{x^2+1} - \frac{6x+1}{x^2+1}$$

Add or subtract. Identify any x-values for which the expression is undefined.

$$\frac{6x + 5}{x^2 - 3} + \frac{3x - 1}{x^2 - 3}$$

Add or subtract. Identify any x-values for which the expression is undefined.

$$\frac{3x^2 - 5}{3x - 1} - \frac{2x^2 - 3x - 2}{3x - 1}$$

To find the least common multiple of a polynomial first list out all of its factors and then find the similar factors, list those only once and list all of the other factors. Multiply out...

Look at the next slide for examples

Find the least common multiple for each pair.

A. $4x^2y^3$ **and** $6x^4y^5$

B. $x^2 - 2x - 3$ **and** $x^2 - x - 6$

Find the least common multiple for each pair.

a. $4x^3y^7$ **and** $3x^5y^4$

b. $x^2 - 4$ **and** $x^2 + 5x + 6$

Find the least common multiple for each pair.

a. $4x^3y^7$ **and** $3x^5y^4$

b. $x^2 - 4$ **and** $x^2 + 5x + 6$

To add rational expressions with unlike denominators, rewrite both expressions with the LCD. This process is similar to adding fractions.

First find the least common multiple. Apply that to top and bottom of fraction. Add them and then cancel things before simplifying!

After it is completely simplified is when you find the undefined x-values for the problem.

Add. Identify any *x*-values for which the expression is undefined.

$$\frac{x - 3}{x^2 + 3x - 4} + \frac{2x}{x + 4}$$

Add. Identify any *x*-values for which the expression is undefined.

$$\frac{x}{x + 2} + \frac{-8}{x^2 - 4}$$

Add. Identify any x-values for which the expression is undefined.

$$\frac{x}{x+3} + \frac{2x+6}{x^2+6x+9}$$

Subtract $\frac{2x^2-30}{x^2-9} - \frac{x+5}{x+3}$. Identify any x-values for which the expression is undefined.

Subtract $\frac{3x - 2}{2x + 5} - \frac{2}{5x - 2}$. Identify any x-values for which the expression is undefined.

Subtract $\frac{2x^2 + 64}{x^2 - 64} - \frac{x - 4}{x + 8}$. Identify any x-values for which the expression is undefined.

Homework:

p. 580 #18-20, 24-30(even), 32-34, 45

p. 588 #17-21, 22, 24, 26, 34-40(even)

7-4 Properties of Logarithms

Remember that to *multiply* powers with the same base, you *add* exponents.

$$b^m b^n = b^{m+n}$$

Product Property of Logarithms

For any positive numbers m, n, and b $(b \neq 1)$,

WORDS	NUMBERS	ALGEBRA
The logarithm of a product is equal to the sum of the logarithms of its factors.	$\log_3 1000 = \log_3(10 \cdot 100)$ $= \log_3 10 + \log_3 100$	$\log_b mn = \log_b m + \log_b n$

Holt Algebra 2

7-4 Properties of Logarithms

Quotient Property of Logarithms

For any positive numbers m, n, and b $(b \neq 1)$,

WORDS	NUMBERS	ALGEBRA
The logarithm of a quotient is the logarithm of the dividend minus the logarithm of the divisor.	$\log_5\left(\frac{16}{2}\right) = \log_5 16 - \log_5 2$	$\log_b \frac{m}{n} = \log_b m - \log_b n$

The property above can also be used in reverse.

Caution

Just as a^5b^3 cannot be simplified, logarithms must have the same base to be simplified.

7-4 Properties of Logarithms

Because you can multiply logarithms, you can also take powers of logarithms.

Power Property of Logarithms

For any real number p and positive numbers a and b $(b \neq 1)$,

WORDS	NUMBERS	ALGEBRA
The logarithm of a power is the product of the exponent and the logarithm of the base.	$\log 10^3$ $\log(10 \cdot 10 \cdot 10)$ $\log 10 + \log 10 + \log 10$ $3\log 10$	$\log_b a^p = p\log_b a$

7-4 Properties of Logarithms

Most calculators calculate logarithms only in base 10 or base *e* (see Lesson 7-6). You can change a logarithm in one base to a logarithm in another base with the following formula.

Change of Base Formula

For $a > 0$ and $a \neq 1$ and any base b such that $b > 0$ and $b \neq 1$,

ALGEBRA	EXAMPLE
$\log_b x = \dfrac{\log_a x}{\log_a b}$	$\log_4 8 = \dfrac{\log_2 8}{\log_2 4}$

We have found a logarithmic property that says:

$\log_a b^x =$

So if we had an equation we could use that same property to help isolate the x. REMEMBER: if you take the log of one side you have to take the log of the other side...

Ex: $4^x = 56$ solve for x.

Ex: $3^x = 27^{x-2}$

27 is just 3^3 so we could switch 27 with 3^3 to get $3^x = (3^3)^{x-2}$.

We then use our exponent properties to get $3^x = 3^{3x-6}$.

Now since our bases are equal we can set the powers equal to each other and solve.

$x = 3x - 6$

Note!!!!

If x is in the exponent on both sides of the equation get the same base.

If x is only in one exponent then take the log of both sides!

Steps for when x is in the logarithm:
1) Simplify each side as much as possible using logarithm rules.
2) Use the hook rule to write the logarithm as an exponent and get rid of it
3) Solve for x