Warm Up:

Solve each equation.

1. $\frac{2.4}{x} = \frac{2}{9}$ $\quad \frac{2x}{2} = \frac{21.6}{2}$ $\quad x = 10.8$

2. $1.6x = 1.8(24.8)$

Determine whether each data set could represent a linear function.

3.

x	2	4	6	8
y	12	6	4	3

x: +2, +2, +2; y: −6, −2, −1

Not Linear

4.

x	−2	−1	0	1
y	−6	−2	2	6

x: +1, +1, +1; y: +4, +4, +4

Linear

8-1 Variation Functions

Objective

Solve problems involving direct, inverse, joint, and combined variation.

Holt Algebra 2

8-1 Variation Functions

Vocabulary

direct variation
constant of variation
joint variation
inverse variation
combined variation

8-1 Variation Functions

In Chapter 2, you studied many types of linear functions. One special type of linear function is called *direct variation*. A **direct variation** is a relationship between two variables x and y that can be written in the form $y = kx$, where $k \neq 0$. In this relationship, k is the **constant of variation**. For the equation $y = kx$, y varies directly as x.

$y = mx + b \leftarrow 0$

$y = mx$

Holt Algebra 2
Copyright © by Holt, Rinehart and Winston. All Rights Reserved.

8-1 Variation Functions

A direct variation equation is a linear equation in the form $y = mx + b$, where $b = 0$ and the constant of variation k is the slope. Because $b = 0$, the graph of a direct variation always passes through the origin.

y=kx

y=kx

With a direct variation we are using the equation:

$$y = kx$$

To solve these problems:

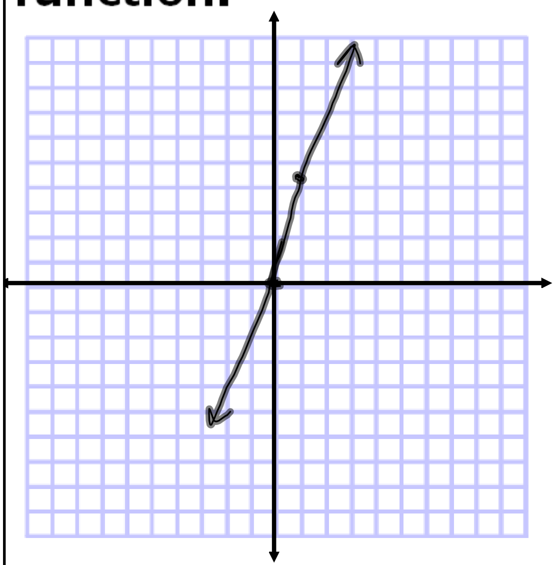
1) Find k by plugging in values for x and y.

2) Find the missing value using substitution again.

Like said earlier, all y = kx is the y=mx+b line with a y-intercept of 0. So the k value is just the slope of a linear function.

$k \rightarrow$ slope

Given: ***y*** **varies directly as** ***x*****, and** ***y*** **= 27 when** ***x*** **= 6. Write and graph the direct variation function.**



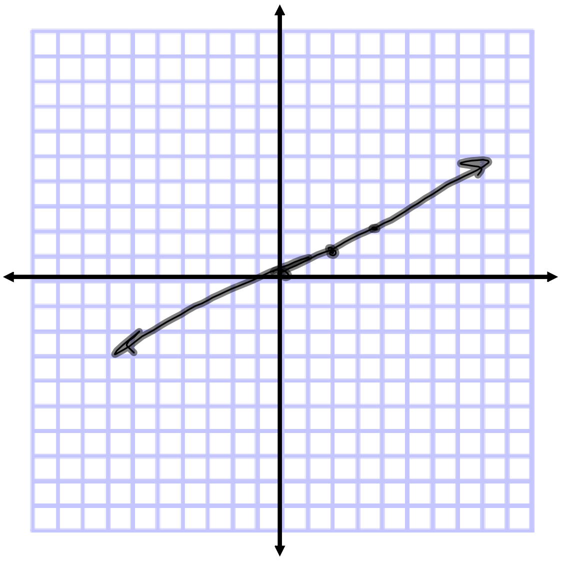
$y = kx$

$\frac{27}{6} = \frac{k \cdot 6}{6}$

$k = 4.5$

$y = 4.5x$

Given: *y* varies directly as *x*, and *y* = 6.5 when *x* = 13. Write and graph the direct variation function.



$y = kx$

$\frac{6.5}{13} = \frac{k \cdot 13}{13}$

$k = \frac{1}{2}$

$y = \frac{1}{2}x$

8-1 Variation Functions

When you want to find specific values in a direct variation problem, you can solve for *k* and then use substitution or you can use the proportion derived below.

12/40

When you want to find specific values in a direct variation problem, you can solve for k and then use substitution or you can use the proportion derived below.

$$y_1 = kx_1 \rightarrow \frac{y_1}{x_1} = k \quad \text{and} \quad y_2 = kx_2 \rightarrow \frac{y_2}{x_2} = k \quad \text{so,} \quad \frac{y_1}{x_1} = \frac{y_2}{x_2}.$$

The cost of an item in euros *e* varies directly as the cost of the item in dollars *d*, and *e* = 3.85 euros when *d* = \$5.00. Find *d* when *e* = 10.00 euros.

$e = kd$

$\frac{3.85}{5} = \frac{k \cdot 5}{5}$

$k = .77$

$e = .77d$

$\frac{10}{.77} = \frac{.77d}{.77}$

$d = 12.98$

The perimeter *P* of a regular dodecagon varies directly as the side length *s*, and *P* = 18 in. when *s* = 1.5 in. Find *s* when *P* = 75 in.

$P = ks$

$\frac{18}{1.5} = \frac{k \cdot 1.5}{1.5}$

$k = 12$

$\frac{75}{12} = \frac{12s}{12}$

$s = 6.25$ in

A **joint variation** is a relationship among three variables that can be written in the form $y = kxz$, where k is the constant of variation. For the equation $y = kxz$, y varies jointly as x and z.

Adding an extra variable doesn't change how we solve these problems.
Steps:
1) solve y=kxz
2) use the value of k to find the missing value.

The volume *V* of a cone varies jointly as the area of the base *B* and the height *h*, and *V* = 12π ft³ when *B* = 9π ft³ and *h* = 4 ft. Find *b* when *V* = 24π ft³ and *h* = 9 ft.

$V = kBh$

$12 = k \cdot 9 \cdot 4$

$\frac{12}{36} = \frac{36k}{36}$

$k = \frac{1}{3}$

$V = \frac{1}{3}Bh$

$24 = \frac{1}{3} \cdot B \cdot 9$

$\frac{24}{3} = \frac{3B}{3}$

$B = 8$

The lateral surface area *L* of a cone varies jointly as the area of the base radius *r* and the slant height *l*, and *L* = 63π m² when *r* = 3.5 m and *l* = 18 m. Find *r* to the nearest tenth when *L* = 8π m² and *l* = 5 m.

$L = krl$

$63\pi = k \cdot 3.5 \cdot 18$

$\frac{63\pi}{63} = \frac{63k}{63}$

$k = \pi$

$L = \pi r l$

$8\pi = \pi r \cdot 5$

$\frac{8\pi}{5\pi} = \frac{5\pi r}{5\pi}$

$r = \frac{8}{5} = 1.6$

8-1 Variation Functions

A third type of variation describes a situation in which one quantity increases and the other decreases. For example, the table shows that the time needed to drive 600 miles decreases as speed increases.

Speed (mi/h)	Time (h)	Distance (mi)
30	20	600
40	15	600
50	12	600

This type of variation is an inverse variation. An **inverse variation** is a relationship between two variables x and y that can be written in the form $y = \frac{k}{x}$, where $k \neq 0$. For the equation $y = \frac{k}{x}$, y varies inversely as x.

Holt Algebra 2

$Y = \frac{k}{X}$

Inverse variation works exactly the same as direct variation except now the formula is

$$y = k/x$$

1) Find k
2) Use substitution to find missing value.

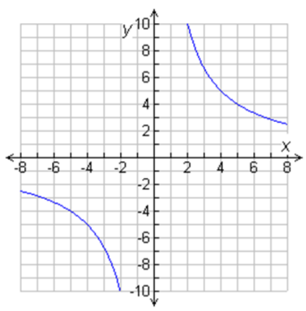
8-1 Variation Functions

Example 4 Continued

To graph, make a table of values for both positive and negative values of x. Plot the points, and connect them with two smooth curves. Because division by 0 is undefined, the function is undefined when $x = 0$.

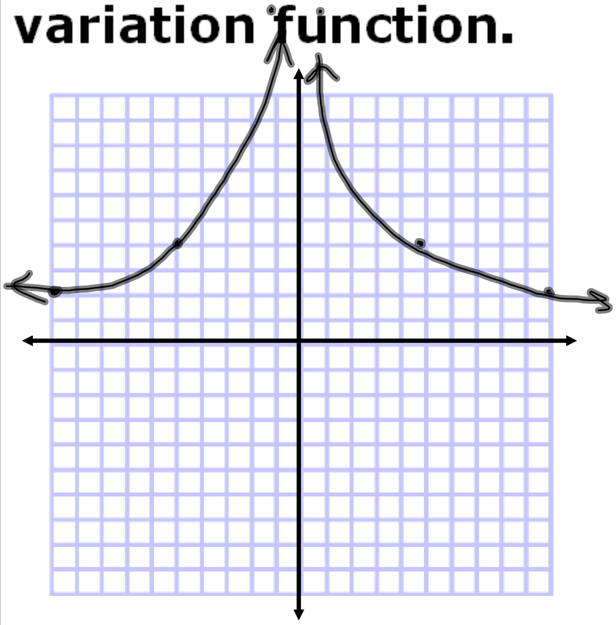
x	y
–2	–10
–4	–5
–6	$-\frac{10}{3}$
–8	$-\frac{5}{2}$

x	y
2	10
4	5
6	$\frac{10}{3}$
8	$\frac{5}{2}$



Holt Algebra 2

Given: *y* varies inversely as *x*, and *y* = 4 when *x* = 5. Write and graph the inverse variation function.



$y = \frac{k}{x}$

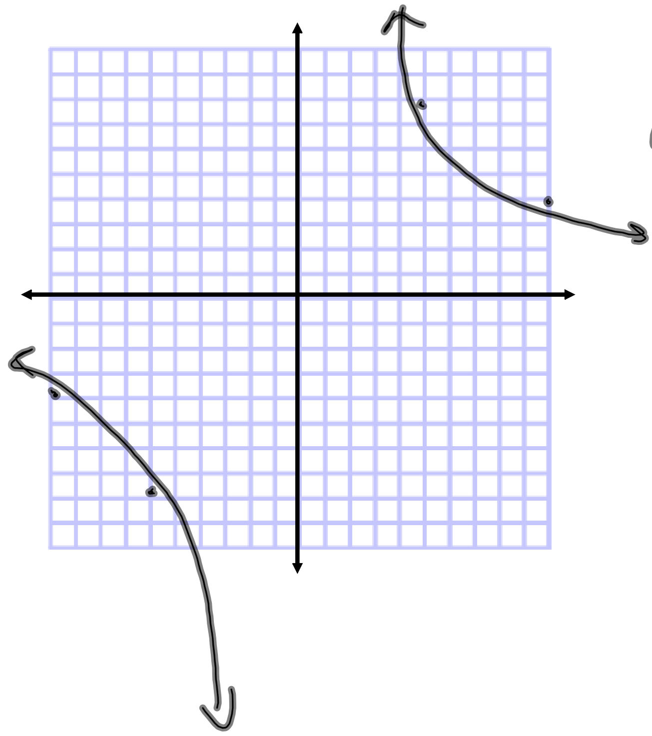
$4 = \frac{k}{5}$

$k = 20$

$y = \frac{20}{x}$

x	y
-10	-2
-5	-4
-1	-20
1	20
5	4
10	2

Given: *y* varies inversely as *x*, and *y* = 4 when *x* = 10. Write and graph the inverse variation function.



$y = \frac{k}{x}$

$4 = \frac{k}{10}$

$k = 40$

$y = \frac{40}{x}$

x	y
-10	-4
-5	-8
5	8
10	4

The time *t* needed to complete a certain race varies inversely as the runner's average speed *s*. If a runner with an average speed of 8.82 mi/h completes the race in 2.97 h, what is the average speed of a runner who completes the race in 3.5 h?

$t = \frac{k}{s}$

$8.82 \cdot 2.97 = \frac{k}{8.82} \cdot 8.82$

$k = 26.2$

$t = \frac{26.2}{s}$

$s \cdot 3.5 = \frac{26.2}{s} \cdot s$

$\frac{3.5s}{3.5} = \frac{26.2}{3.5}$

$s = 7.49 \frac{mi}{hr}$

The time *t* that it takes for a group of volunteers to construct a house varies inversely as the number of volunteers *v*. If 20 volunteers can build a house in 62.5 working hours, how many working hours would it take 15 volunteers to build a house?

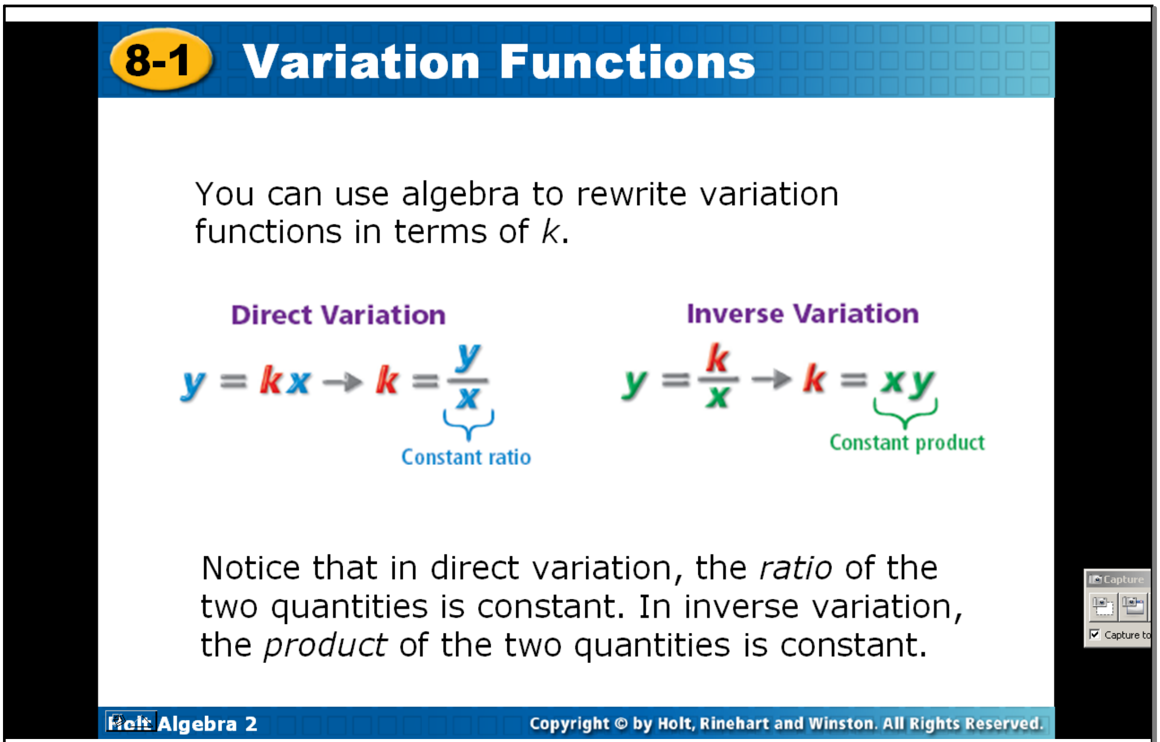
$t = \frac{k}{v}$

$62.5 = \frac{k}{20}$

$k = 1250$

$t = \frac{1250}{15}$

$t = 83.3$ hr



To find out if a table of values if direct or inverse check the following:

1) Is y/x the same for each consecutive set of points? If yes then the table represents a direct variation. $\frac{y}{x}$

2) Is yx the same for each set of points? If yes then the table is an inverse variation. xy

Note: If neither, just say neither

Determine whether each data set represents a direct variation, an inverse variation, or neither.

A.

x	6.5	13	104
y	8	4	0.5

→ inverse Variation

$6.5 \cdot 8 = 52$ $13 \cdot 4 = 52$ $104 \cdot .5 = 52$

B.

x	5	8	12
y	30	48	72

→ Direct Variation

$30 \cdot 5 = 150$

$8 \cdot 48 = 384$

$\frac{30}{5} = 6$

$\frac{48}{8} = 6$

$\frac{72}{12} = 6$

Determine whether each data set represents a direct variation, an inverse variation, or neither.

C.

x	3	6	8
y	5	14	21

→ Neithe

$\frac{5}{3} = 1.\overline{6}$

$\frac{14}{6} = 2.\overline{6}$

$3 \cdot 5 = 15$

$6 \cdot 14 = 84$

Determine whether each data set represents a direct variation, an inverse variation, or neither.

6a.

x	3.75	15	5
y	12	3	9

6b.

x	1	40	26
y	0.2	8	5.2

8-1 Variation Functions

A **combined variation** is a relationship that contains both direct and inverse variation. Quantities that vary directly appear in the numerator, and quantities that vary inversely appear in the denominator.

$y = kxz$

$y = \frac{kx}{z}$

Holt Algebra 2

For combined variations just set up the equation and solve for k, then use k to find the missing value.

The change in temperature of an aluminum wire varies inversely as its mass *m* and directly as the amount of heat energy *E* transferred. The temperature of an aluminum wire with a mass of 0.1 kg rises 5°C when 450 joules (J) of heat energy are transferred to it. How much heat energy must be transferred to an aluminum wire with a mass of 0.2 kg raise its temperature 20°C?

$t = \frac{kE}{m}$

$5 = \frac{450k}{.1}$

$\frac{5}{450} = \frac{4500k}{4500}$

$k = .001$

$.2 \cdot 20 = \frac{.001E}{.2} \cdot .2$

$\frac{.4}{.001} = \frac{.001E}{.001}$

$E = 400 \cancel{J}$

The volume *V* of a gas varies inversely as the pressure *P* and directly as the temperature *T*. A certain gas has a volume of 10 liters (L), a temperature of 300 kelvins (K), and a pressure of 1.5 atmospheres (atm). If the gas is heated to 400K, and has a pressure of 1 atm, what is its volume?

$V = \frac{kT}{P}$

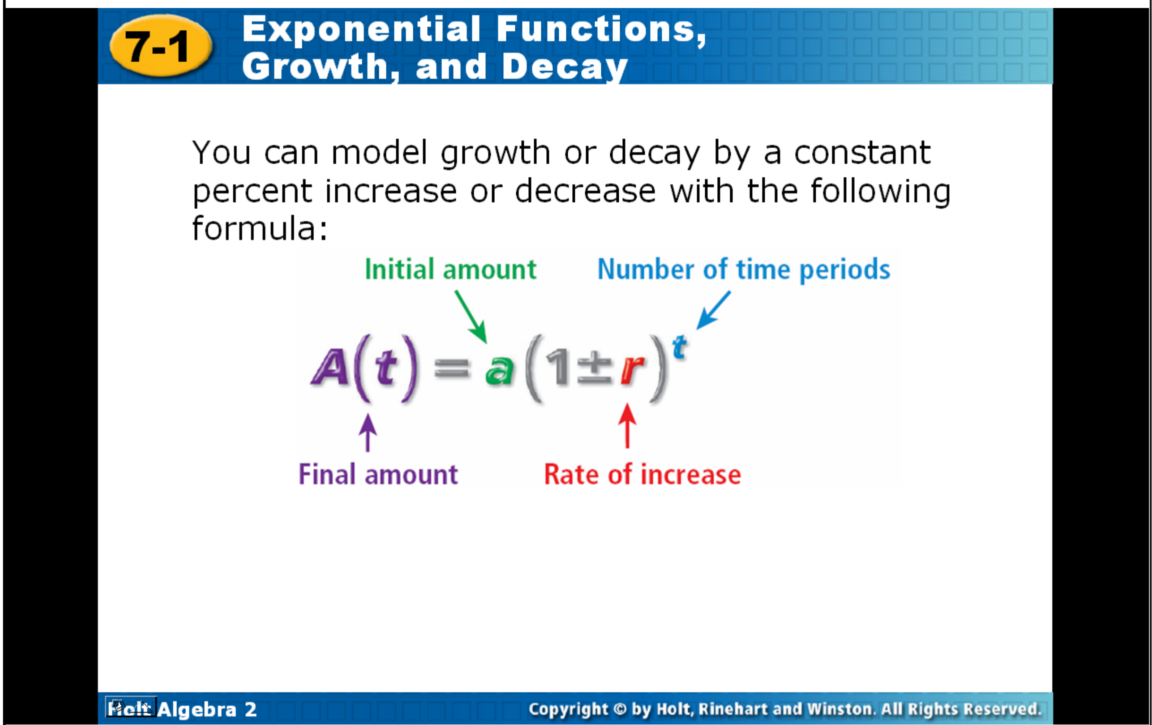
$10 = \frac{k \cdot 300}{1.5}$

$k = .05$

$V = \frac{.05T}{P}$

$V = \frac{.05 \cdot 400}{1}$

$V = 20$atm



7-1 Exponential Functions, Growth, and Decay

Example 2: Economics Application

Clara invests $5000 in an account that pays 6.25% interest per year. After how many years will her investment be worth $10,000?

7-2 Inverses of Relations and Functions

You can also find and apply inverses to relations and functions. To graph the **inverse relation**, you can reflect each point across the line $y = x$. This is equivalent to switching the x- and y-values in each ordered pair of the relation.

Remember!

A *relation* is a set of ordered pairs. A *function* is a relation in which each x-value has, at most, one y-value paired with it.

All we do for inverses is switch x and y, then solve for y. Don't switch numbers, don't switch signs, just x and y.

Find the inverse of y = 2x - 7

$$x = 2y - 7$$

$$x + 7 = 2y$$

$$y = \frac{x+7}{2}$$

7-3 Logarithmic Functions

Example 2: Converting from Logarithmic to Exponential Form

Write each logarithmic form in exponential equation.

Logarithmic Form	Exponential Equation
$\log_9 9 = 1$	$9^1 = 9$
$\log_2 512 = 9$	$2^9 = 512$
$\log_8 2 = \frac{1}{3}$	$8^{\frac{1}{3}} = 2$
$\log_4 \frac{1}{16} = -2$	
$\log_b 1 = 0$	

Holt Algebra 2

7-3 Logarithmic Functions

Example 1: Converting from Exponential to Logarithmic Form

Write each exponential equation in logarithmic form.

Exponential Equation	Logarithmic Form
$3^5 = 243$	$\log_3 243 = 5$
$25^{\frac{1}{2}} = 5$	
$10^4 = 10{,}000$	
$6^{-1} = \frac{1}{6}$	
$a^b = c$	

$3^5 = 243$

7-3 Logarithmic Functions

Example 3B: Evaluating Logarithms by Using Mental Math

Evaluate by using mental math.

$\log_5 125 = x = 3$

$5^x = 125$

$x = 3$

Homework:

p. 574 # 17-23, 31, 40-41, 43