Warm up:
Write the following in exponential form:

$x = \log_4 52$ $\log x = 7$ $\log_m n = y$

$4^x = 52$ $10^7 = x$ $m^y = n$

Write the following in logarithmic form:

$3^x = 28$ $y^5 = 50$ $c^t = m$

$\log_3 28 = x$

Just like exponents, logarithms have special properties for simplification. Work in partners/groups to find these properties...

$x^2 \cdot x^5$

x^7

$\log 9 + \log 3$

$\log 27$

7-4 Properties of Logarithms

Objectives

Use properties to simplify logarithmic expressions.

Translate between logarithms in any base.

7-4 Properties of Logarithms

Remember that to *multiply* powers with the same base, you *add* exponents.

$b^m b^n = b^{m+n}$

$\log 4 + \log 5 = \log 20$

Product Property of Logarithms

For any positive numbers m, n, and b $(b \neq 1)$,

WORDS	NUMBERS	ALGEBRA
The logarithm of a product is equal to the sum of the logarithms of its factors.	$\log_3 1000 = \log_3(10 \cdot 100)$ $= \log_3 10 + \log_3 100$	$\log_b mn = \log_b m + \log_b n$

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7-4 Properties of Logarithms

Example 1: Adding Logarithms

Express $\log_6 4 + \log_6 9$ as a single logarithm. Simplify.

$\log_6 36$

$\log_6 36 = x$

$6^x = 36$

2

7-4 Properties of Logarithms

Check It Out! Example 1a

Express as a single logarithm. Simplify, if possible.

$\log_5 625 + \log_5 25$

$\log_5 15625$

7-4 Properties of Logarithms

Check It Out! Example 1b

Express as a single logarithm. Simplify, if possible.

$$\log_{\frac{1}{3}} 27 + \log_{\frac{1}{3}} \frac{1}{9}$$

$\log_{\frac{1}{3}} 3 = x$

$\left(\frac{1}{3}\right)^x = 3$

7-4 Properties of Logarithms

$\log 40 - \log 10 = \log 4$

Quotient Property of Logarithms

For any positive numbers m, n, and b $(b \neq 1)$,

WORDS	NUMBERS	ALGEBRA
The logarithm of a quotient is the logarithm of the dividend minus the logarithm of the divisor.	$\log_5\left(\frac{16}{2}\right) = \log_5 16 - \log_5 2$	$\log_b \frac{m}{n} = \log_b m - \log_b n$

The property above can also be used in reverse.

Caution

Just as a^5b^3 cannot be simplified, logarithms must have the same base to be simplified.

7-4 Properties of Logarithms

Example 2: Subtracting Logarithms

Express $\log_5 100 - \log_5 4$ as a single logarithm. Simplify, if possible.

$\log_5 25$

$\log_5 25 = x$ 2

$5^x = 25$

7-4 Properties of Logarithms

Check It Out! Example 2

Express $\log_7 49 - \log_7 7$ as a single logarithm. Simplify, if possible.

$\log_7 7$

1

7-4 Properties of Logarithms

$\log 4^3 = 3 \cdot \log 4$

Because you can multiply logarithms, you can also take powers of logarithms.

Power Property of Logarithms

For any real number p and positive numbers a and b $(b \neq 1)$,

WORDS	NUMBERS	ALGEBRA
The logarithm of a power is the product of the exponent and the logarithm of the base.	$\log 10^3$ $\log(10 \cdot 10 \cdot 10)$ $\log 10 + \log 10 + \log 10$ $3 \log 10$	$\log_b a^p = p \log_b a$

7-4 Properties of Logarithms

Example 3: Simplifying Logarithms with Exponents

Express as a product. Simplify, if possible.

A. $\log_2 32^6$

$6 \cdot \log_2 32$

$6 \cdot 5 = 30$

$\log_2 32 = x$

$2^x = 32$

$x = 5$

B. $\log_8 4^{20}$

$\log_8 4$

$8^x = 4$

$20 \cdot \log_8 4$

7-4 Properties of Logarithms

Check It Out! Example 3

Express as a product. Simplify, if possibly.

a. $\log 10^4$

$4 \cdot \log 10 = x$

$4 \cdot 1$

4

b. $\log_5 25^2$

$2 \cdot \log_5 25 = x$

$2 \cdot 2$

4

7-4 Properties of Logarithms

Check It Out! Example 3

Express as a product. Simplify, if possibly.

c. $\log_2 \left(\frac{1}{2}\right)^5$

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7-4 Properties of Logarithms

Exponential and logarithmic operations undo each other since they are inverse operations.

Inverse Properties of Logarithms and Exponents

For any base b such that $b > 0$ and $b \neq 1$,

ALGEBRA	EXAMPLE
$\log_b b^x = x$	$\log_{10} 10^7 = 7$
$b^{\log_b x} = x$	$10^{\log_{10} 2} = 2$

$2^{\log_2 4}$ $\quad$ $\log_2 2^x$

4 $\quad$ x

7-4 Properties of Logarithms

Example 4: Recognizing Inverses

$3^x = 81$

Simplify each expression.

a. $\log_3 3^{11}$

11

b. $\log_3 81 = x$ $\quad$ $3^4 = 81$

$\log_3 3^4$

4

c. $5^{\log_5 10}$

10

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7-4 Properties of Logarithms

Most calculators calculate logarithms only in base 10 or base *e* (see Lesson 7-6). You can change a logarithm in one base to a logarithm in another base with the following formula.

Change of Base Formula

For $a > 0$ and $a \neq 1$ and any base b such that $b > 0$ and $b \neq 1$,

ALGEBRA	EXAMPLE
$\log_b x = \frac{\log_a x}{\log_a b}$	$\log_4 8 = \frac{\log_2 8}{\log_2 4}$

7-4 Properties of Logarithms

Example 5: Changing the Base of a Logarithm

Evaluate $\log_{32} 8$.

Method 1 Change to base 10

$$\frac{\log_{10} 8}{\log_{10} 32}$$

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7-4 Properties of Logarithms

Check It Out! Example 5a

Evaluate $\log_9 27$.

Method 1 Change to base 10.

$$\frac{\log_{10} 27}{\log_{10} 9}$$

7-4 Properties of Logarithms

Check It Out! Example 5b

Evaluate $\log_8 16$.

Method 1 Change to base 10.

Write out your own simplification problems. Challenge your partners to solve them.

Find my mistake...

log 3 + log 7 = log 10 log 21

$\log_8 54 - \log_4 9 = \log 6$ 2 log 24

2 (log 4 + log 6) = log 96

log 15 - log 3 + log 4 = log 1.25

log 5 + log 4 = log 20

Homework
p. 516 #20-34, 51-53,
60, 62, 65

60 65