Warm Up:

Write a quadratic function in standard form with zeros 6 and −1.

$(x-6)(x+1)$

x^2-5x-6

A rocket is launched from ground level with an initial vertical velocity of 176 ft/s. After how many seconds with the rocket hit the ground?

$-16t^2+176t=0$

$16t(-t^2+11)=0$

$t=0,11$

We will start this lesson off with a video...

5-5 Complex Numbers and Roots

Objectives

Define and use imaginary and complex numbers.

Solve quadratic equations with complex roots.

5-5 Complex Numbers and Roots

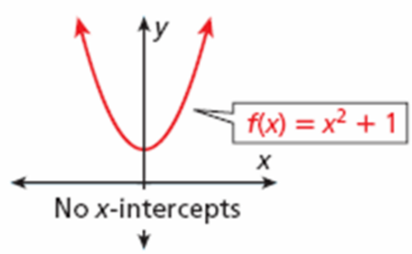
Vocabulary

imaginary unit
imaginary number
complex number
real part
imaginary part
complex conjugate

5-5 Complex Numbers and Roots

You can see in the graph of $f(x) = x^2 + 1$ below that f has no real zeros. If you solve the corresponding equation $0 = x^2 + 1$, you find that $x = \pm\sqrt{-1}$,which has no *real* solutions.

However, you can find solutions if you define the square root of negative numbers, which is why *imaginary numbers* were invented. The **imaginary unit** *i* is defined as $\sqrt{-1}$. You can use the imaginary unit to write the square root of any negative number.



5-5 Complex Numbers and Roots

Imaginary Numbers

WORDS	NUMBERS	ALGEBRA
An **imaginary number** is the square root of a negative number. Imaginary numbers can be written in the form *bi*, where *b* is a real number and *i* is the imaginary unit. The square of an imaginary number is the original negative number.	$\sqrt{-1} = i$ $\sqrt{-2} = \sqrt{-1}\sqrt{2} = i\sqrt{2}$ $\sqrt{-4} = \sqrt{-1}\sqrt{4} = 2i$ $\left(\sqrt{-1}\right)^2 = i^2 = -1$	If b is a positive real number, then $\sqrt{-b} = i\sqrt{b}$ and $\sqrt{-b^2} = bi$. $\left(\sqrt{-b}\right)^2 = -b$

Factor out what you can, then replace $\sqrt{-1}$ with i and work from there.

Express the number in terms of *i*.

$5\sqrt{-121}$

$5 \cdot \sqrt{121} \cdot \sqrt{-1}$

$5 \cdot 11 \cdot i$

$55i$

Express the number in terms of *i*.

$-\sqrt{-96}$

$-\sqrt{16}\cdot\sqrt{6}\cdot\sqrt{-1}$

$-4\sqrt{6}\,i$

Express the number in terms of *i*.

$\sqrt{-12}$

$\sqrt{4}\cdot\sqrt{3}\cdot\sqrt{-1}$

$2\sqrt{3}\,i$

Express the number in terms of *i*.

$2\sqrt{-36}$

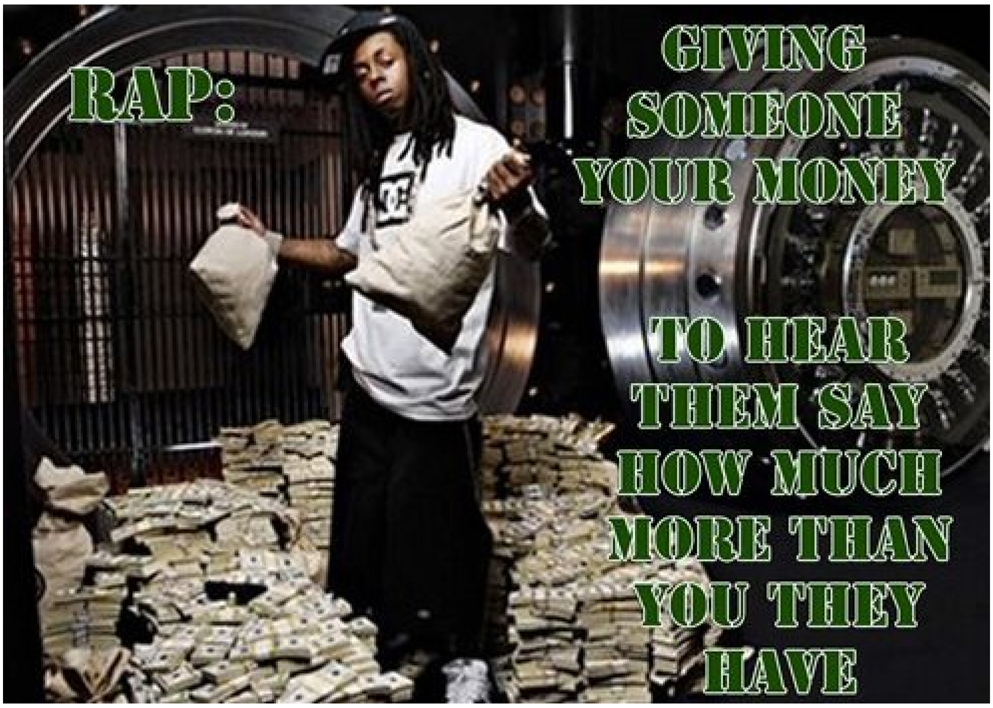
$2\sqrt{36}\cdot\sqrt{-1}$

$2\cdot 6i$

$12i$

Express the number in terms of *i*.

$-\frac{1}{3}\sqrt{-63}$



Solving equations works similar, get the variable by itself and if you have the square root of a negative number, remember i is useful.

Solve the equation.

$$5x^2 + 90 = 0$$

$$-90 \quad -90$$

$$\frac{5x^2}{5} = \frac{-90}{5}$$

$$\sqrt{x^2} = \sqrt{-18}$$

$$x = \pm\sqrt{-18}$$

$$x = \pm\sqrt{9}\cdot\sqrt{2}\cdot\sqrt{-1}$$

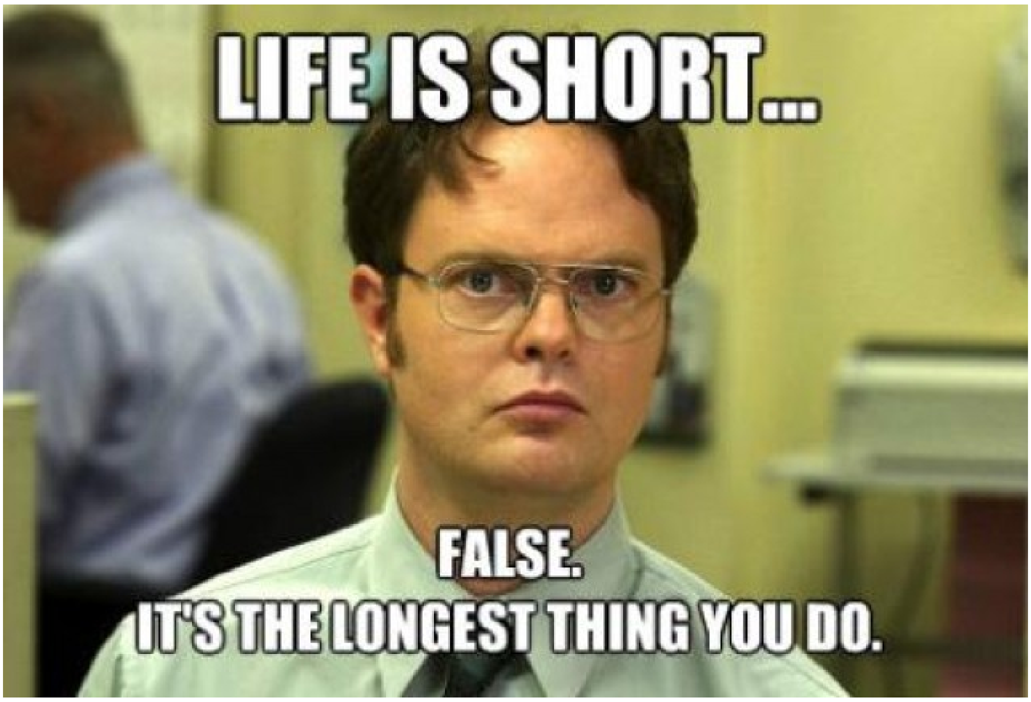
$$x = \pm 3\sqrt{2}i$$

Solve the equation.

$$x^2 = -144$$

$$x = \pm\sqrt{-144} = \sqrt{144}\cdot\sqrt{-1}$$

$$x = \pm 12i$$



Solve the equation.

$x^2 = -36$

$x = \pm\sqrt{-36}$

$x = \pm 6i$

Solve the equation.

$x^2 + 48 = 0$

Solve the equation.

$9x^2 + 25 = 0$

$-25 \quad -25$

$\frac{9x^2}{9} = \frac{-25}{9}$

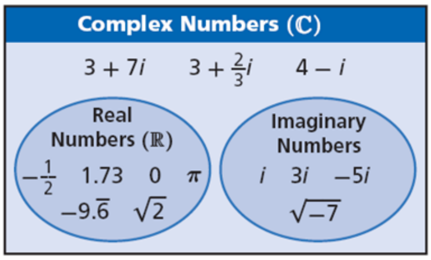
$\sqrt{x^2} = \sqrt{\frac{-25}{9}}$

$x = \pm\sqrt{\frac{-25}{9}}$

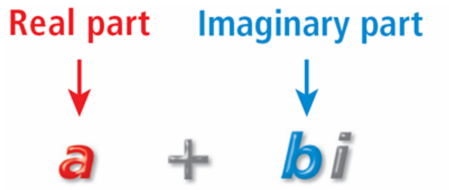
$x = \pm\frac{5}{3}i$

5-5 Complex Numbers and Roots

A **complex number** is a number that can be written in the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$. The set of real numbers is a subset of the set of complex numbers C.



Every complex number has a **real part** a and an **imaginary part** b.



5-5 Complex Numbers and Roots

Real numbers are complex numbers where $b = 0$. Imaginary numbers are complex numbers where $a = 0$ and $b \neq 0$. These are sometimes called *pure imaginary numbers*.

Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal.

To find equal values, set the real part of the complex number equal to the other, and set the imaginary part equal to the other. You should have 2 equations.

Find the values of *x* and *y* that make the equation $4x + 10i = 2 - (4y)i$ true .

$4x = 2$

$x = \frac{1}{2}$

$10i = -4yi$

$y = \frac{-5}{2}$

Find the values of x and y that make each equation true.

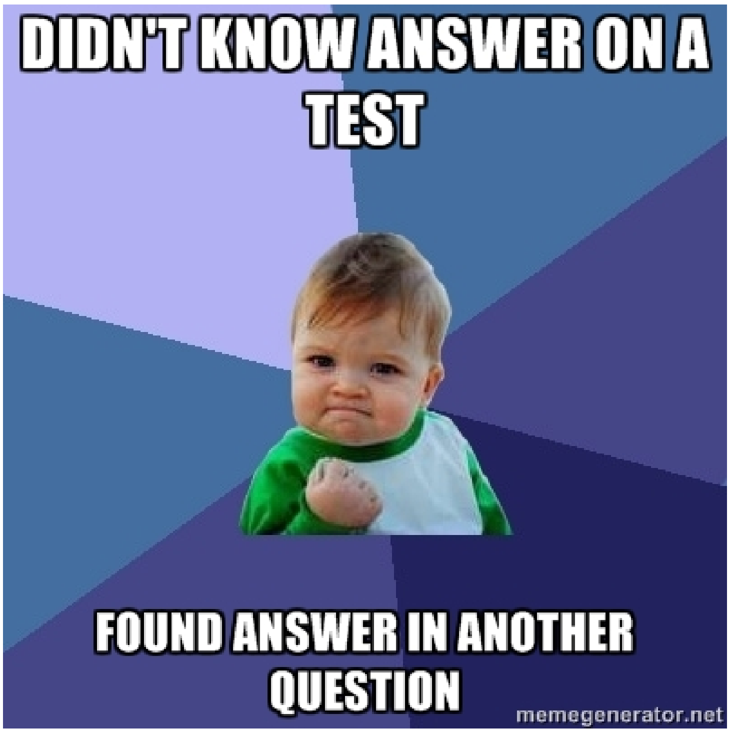
$$2x - 6i = -8 + (20y)i$$

$2x = -8$ $\quad -6 = 20y$

$x = -4$ $\quad y = \frac{-3}{10}$

Find the values of x and y that make each equation true.

$$-8 + (6y)i = 5x - i\sqrt{6}$$



Something we skipped is called completing the square. I won't go over all the steps but to find zeros you will need to know the basic outline. Follow these steps and you will be golden:

1) subtract c from both sides to get an x^2 and x term by itself.
2) Add $(b/2)^2$ to both sides (this allows you to factor easily)
3) Factor your equation
4) Solve for x

Find the zeros of the function.

$f(x) = x^2 + 10x + 26 = 0$

$-26 \quad -26$

$\left(\frac{10}{2}\right)^2 = 25$

$x^2 + 10x = -26$

$x^2 + 10x + 25 = -26 + 25$

$\sqrt{(x+5)^2} = \sqrt{-1}$

$x + 5 = \pm\sqrt{-1}$

$x = -5 \pm \sqrt{-1}$

$x = -5 \pm i$

Find the zeros of the function.

$g(x) = x^2 + 4x + 12 = 0$

$-12 \quad -12$

$\left(\frac{4}{2}\right)^2 = 4$

$x^2 + 4x = -12$

$x^2 + 4x + 4 = -12 + 4$

$\sqrt{-8} = \sqrt{4} \cdot \sqrt{2} \cdot \sqrt{-1}$

$2\sqrt{2}i$

$(x+2)^2 = -8$

$x + 2 = \pm\sqrt{-8}$

$x = -2 \pm \sqrt{-8}$

$x = -2 \pm 2\sqrt{2}i$

Find the zeros of the function.

$f(x) = x^2 + 4x + 13$

$\left(\frac{4}{2}\right)^2 = 4$

$x^2 + 4x = -13$

$x^2 + 4x + 4 = -9$

$(x+2)^2 = -9$

$x + 2 = \pm\sqrt{-9}$

$x = -2 \pm \sqrt{-9}$

$x = -2 \pm 3i$

Find the zeros of the function.

$g(x) = x^2 - 8x + 18$

5-5 Complex Numbers and Roots

The solutions $-5 + i\sqrt{10}$ and $-5 - i\sqrt{10}$ are related. These solutions are a *complex conjugate* pair. Their real parts are equal and their imaginary parts are opposites. The **complex conjugate** of any complex number $a + bi$ is the complex number $a - bi$.

If a quadratic equation with real coefficients has nonreal roots, those roots are complex conjugates.

Helpful Hint

When given one complex root, you can always find the other by finding its conjugate.

Find each complex conjugate.

A. $8 + 5i$

$8-5i$

B. $6i$

$-6i$

Find each complex conjugate.

A. $9 - i$

$9+i$

B. $i + \sqrt{3}$

$-i+\sqrt{3}$

C. $-8i$

$8i$

The best way to handle this stuff is to practice and to ask questions. Come see me if you are lost. This is important and is used much in pre-calc and calc, and may show up on the ACT. Be sure to stay on top of what we are studying and not walk into class on quiz/test day with no idea of what is happening.

Homework:

p. 353 #18-35, 46-56(even), 76-80