Warm Up:

Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph $g(x) = -\frac{1}{4}(x + 1)^2$.

Reflect over x

V.c $\frac{1}{4}$

Left 1

5-2 **Properties of Quadratic Functions in Standard Form**

Objectives

Define, identify, and graph quadratic functions.

Identify and use maximums and minimums of quadratic functions to solve problems.

5-2 Properties of Quadratic Functions in Standard Form

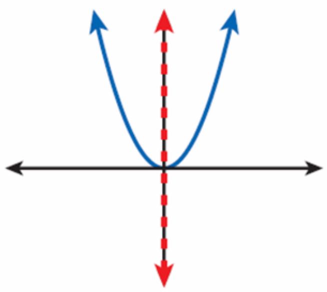
Vocabulary

axis of symmetry
standard form
minimum value
maximum value

5-2 Properties of Quadratic Functions in Standard Form

When you transformed quadratic functions in the previous lesson, you saw that reflecting the parent function across the *y*-axis results in the same function.

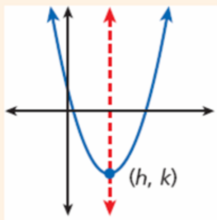
$f(x) = x^2$
$g(x) = (-x)^2 = x^2$



5-2 Properties of Quadratic Functions in Standard Form

This shows that parabolas are symmetric curves. The **axis of symmetry** is the line through the vertex of a parabola that divides the parabola into two congruent halves.

Axis of Symmetry | Quadratic Functions

WORDS	ALGEBRA	GRAPH
The axis of symmetry is a vertical line through the vertex of the function's graph.	The quadratic function $f(x) = a(x - h)^2 + k$ has the axis of symmetry $x = h$.	

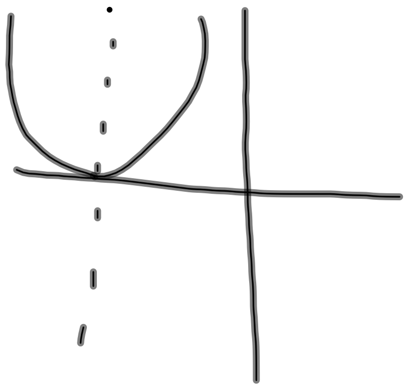
The axis of symmetry is what cuts the parabola in half, so search for the vertex and then use that to find your vertical line of symmetry.

Identify the axis of symmetry for the graph of

$f(x) = -\frac{1}{2}(x+5)^2 - 8$.

5 left

$x = -5$



Identify the axis of symmetry for the graph of

$f(x) = (x-3)^2 + 1$.

$x = 3$

5-2 Properties of Quadratic Functions in Standard Form

Another useful form of writing quadratic functions is the *standard form*. The **standard form** of a quadratic function is $f(x) = ax^2 + bx + c$, where $a \neq 0$.

The coefficients a, b, and c can show properties of the graph of the function. You can determine these properties by expanding the vertex form.

$f(x) = a(x - h)^2 + k$

$f(x) = a(x^2 - 2xh + h^2) + k$ *Multiply to expand $(x - h)^2$.*

$f(x) = a(x^2) - a(2hx) + a(h^2) + k$ *Distribute a.*

$f(x) = ax^2 + (-2ah)x + (ah^2 + k)$ *Simplify and group terms.*

5-2 Properties of Quadratic Functions in Standard Form

$a = a \qquad -2ah = b \qquad ah^2 + k = c$

$$f(x) = ax^2 + bx + c$$

$a = a$ — a in standard form is the same as in vertex form. It indicates whether a reflection and/or vertical stretch or compression has been applied.

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5-2 Properties of Quadratic Functions in Standard Form

$$a = a \qquad -2ah = b \qquad ah^2 + k = c$$

$$f(x) = ax^2 + bx + c$$

$b = -2ah$ Solving for h gives $h = \frac{b}{-2a} = -\frac{b}{2a}$. Therefore, the axis of symmetry, $x = h$, for a quadratic function in standard form is $x = -\frac{b}{2a}$.

5-2 Properties of Quadratic Functions in Standard Form

$$a = a \qquad -2ah = b \qquad ah^2 + k = c$$

$$f(x) = ax^2 + bx + c$$

$c = ah^2 + k$ Notice that the value of c is the same value given by the vertex form of f when $x = 0$: $f(0) = a(0 - h)^2 + k = ah^2 + k$. So c is the y-intercept.

5-2 Properties of Quadratic Functions in Standard Form

These properties can be generalized to help you graph quadratic functions.

Properties of a Parabola

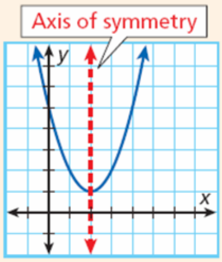
For $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$, the parabola has these properties:

The parabola **opens** upward if $a > 0$ and downward if $a < 0$.

The **axis of symmetry** is the vertical line $x = -\frac{b}{2a}$.

The **vertex** is the point $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

The ***y*-intercept** is c.



Consider the function $f(x) = 2x^2 - 4x + 5$.

a. Determine whether the graph opens upward or downward.

$2 > 0$ up

b. Find the axis of symmetry.

a: 2
b: −4

$x = -\frac{-4}{2(2)}$

$= \frac{4}{4} = 1$

$x = 1$

Consider the function $f(x) = 2x^2 - 4x + 5$.

c. Find the vertex.

(1, 3)

$2(1)^2 - 4(1) + 5$

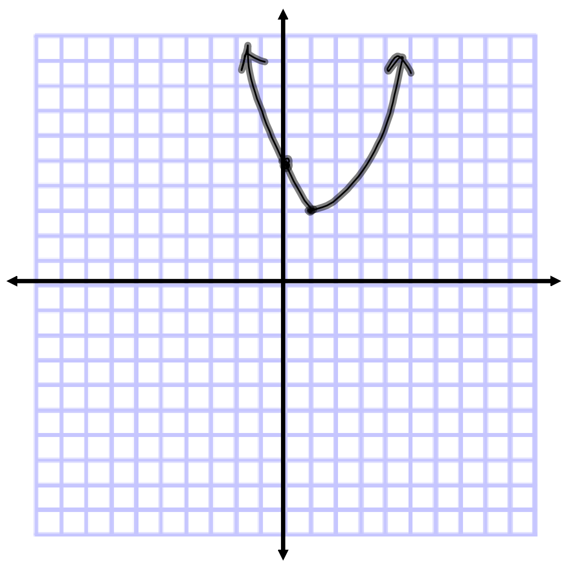
$2 - 4 + 5$

d. Find the y-intercept.

(0, 5)

Consider the function $f(x) = 2x^2 - 4x + 5$.

e. Graph the function.



Consider the function $f(x) = -x^2 - 2x + 3$.

a. Determine whether the graph opens upward or downward.

Down $-1<0$

b. Find the axis of symmetry.

$a: -1$

$b: -2$ $x = -\frac{-2}{2(-1)} = -1$

Consider the function $f(x) = -x^2 - 2x + 3$.

c. Find the vertex.

$(-1, 4)$

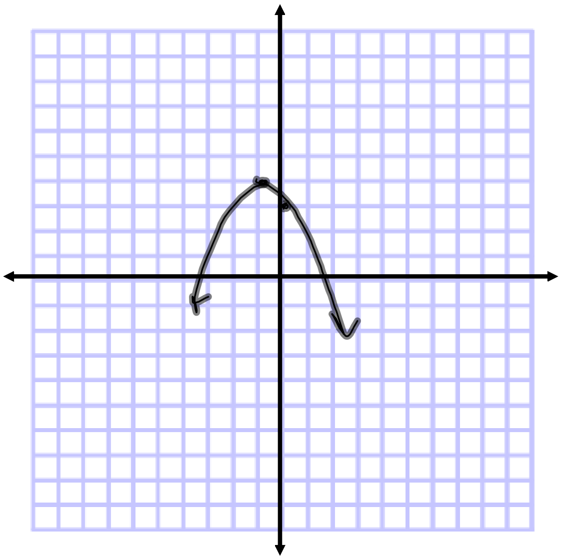
$-(-1)^2 - 2(-1) + 3$

$-1 + 2 + 3$

d. Find the y-intercept.

$(0, 3)$

Consider the function $f(x) = -x^2 - 2x + 3$.
e. Graph the function.



For the function, (a) determine whether the graph opens upward or downward, (b) find the axis of symmetry, (c) find the vertex, (d) find the y-intercept, and (e) graph the function.

$f(x) = -2x^2 - 4x$

For the function, (a) determine whether the graph opens upward or downward, (b) find the axis of symmetry, (c) find the vertex, (d) find the *y*-intercept, and (e) graph the function.

$g(x) = x^2 + 3x - 1.$

a. up

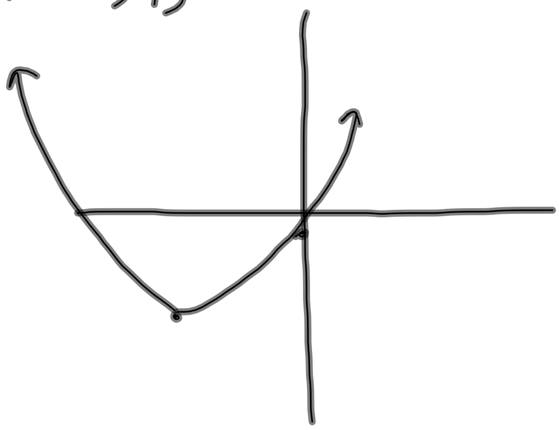
d. (0, -1)

b. a = 1
b = 3

$x = \frac{-3}{2(1)} = -\frac{3}{2}$

-3.25

c. $(-\frac{3}{2}, \cancel{6\frac{3}{4}})$

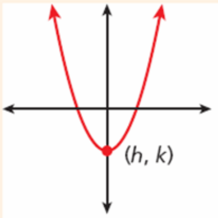
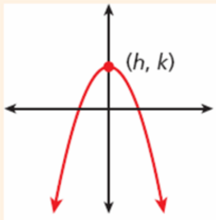


5-2 Properties of Quadratic Functions in Standard Form

Substituting any real value of *x* into a quadratic equation results in a real number. Therefore, the domain of any quadratic function is all real numbers. The range of a quadratic function depends on its vertex and the direction that the parabola opens.

5-2 Properties of Quadratic Functions in Standard Form

Minimum and Maximum Values

OPENS UPWARD	OPENS DOWNWARD
When a parabola opens upward, the y-value of the vertex is the **minimum value**.	When a parabola opens downward, the y-value of the vertex is the **maximum value**.
D: $\{x \mid x \in \mathbb{R}\}$ R: $\{y \mid y \geq k\}$	D: $\{x \mid x \in \mathbb{R}\}$ R: $\{y \mid y \leq k\}$
	
The domain is all real numbers, $\mathbb{R}$. The range is all values greater than or equal to the minimum.	The domain is all real numbers, $\mathbb{R}$. The range is all values less than or equal to the maximum.

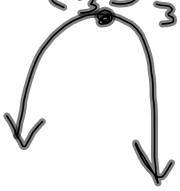
Holt Algebra 2

1) Figure out if your parabola opens up or down, therefore finding a max or min.
2) Find the vertex, which is your max or min
3) Use your vertex and whether it is a max or min to state the ~~domain~~ of the function.
range

Find the minimum or maximum value of $f(x) = -3x^2 + 2x - 4$. Then state the domain and range of the function.

Down

$\left(\frac{1}{3}, -3\frac{2}{3}\right)$



$\frac{-2}{2(-3)} = \frac{1}{3}$

D: $x \in \mathbb{R}$

R: $y \leq -3\frac{2}{3}$

$-3\left(\frac{1}{3}\right)^2 + 2\left(\frac{1}{3}\right) - 4$

$-\frac{1}{3} + \frac{2}{3} - 4$

$-3\frac{2}{3}$

Find the minimum or maximum value of $f(x) = x^2 - 6x + 3$. Then state the domain and range of the function.

$\frac{-(-6)}{2(1)} = 3$

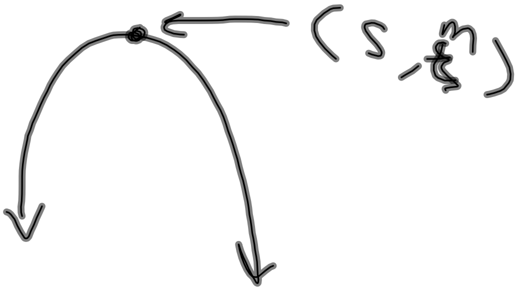
$(3, -6)$

D: $x \in \mathbb{R}$

R: $y \geq -6$

Find the minimum or maximum value of $g(x) = -2x^2 - 4$. Then state the domain and range of the function.

The highway mileage m in miles per gallon for a compact car is approximately by $m(s) = -0.025s^2 + 2.45s - 30$, where s is the speed in miles per hour. What is the maximum mileage for this compact car to the nearest tenth of a mile per gallon? What speed results in this mileage?



Homework:

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