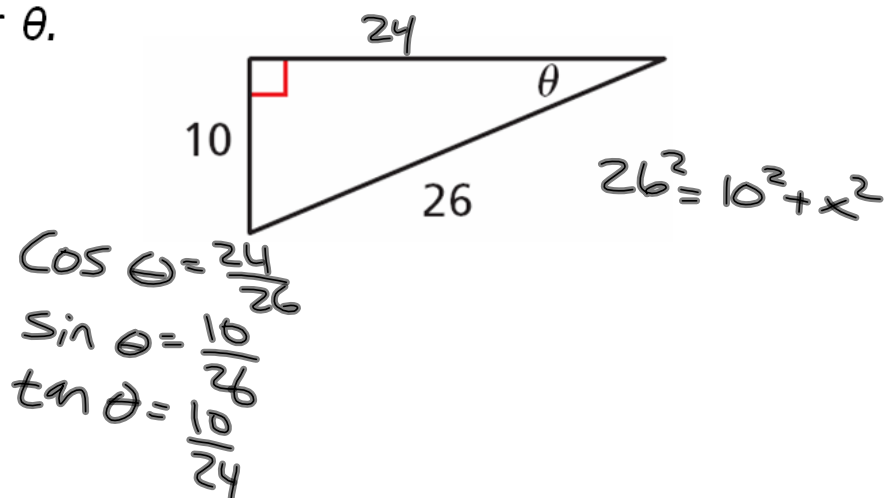


**Warm Up:**

Find the values of the six trigonometric functions for  $\theta$ .

**13-2 Angles of Rotation****Objectives**

Draw angles in standard position.

Determine the values of the trigonometric functions for an angle in standard position.

## 13-2 Angles of Rotation

### *Vocabulary*

standard position

initial side

terminal side

angle of rotation

coterminal angle

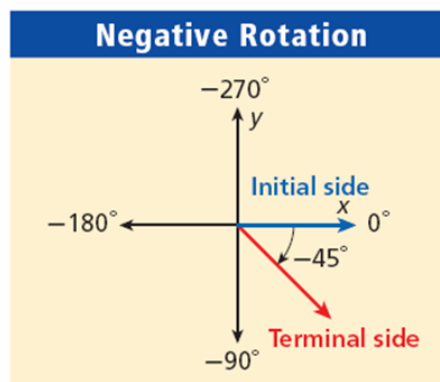
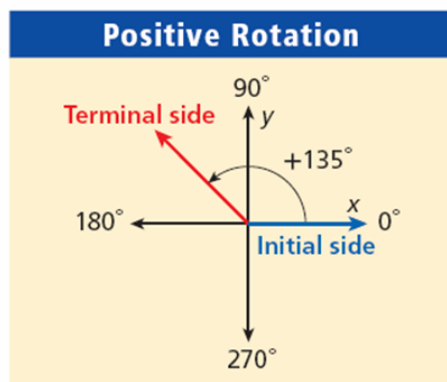
reference angle

## 13-2 Angles of Rotation

In Lesson 13-1, you investigated trigonometric functions by using acute angles in right triangles. The trigonometric functions can also be evaluated for other types of angles.

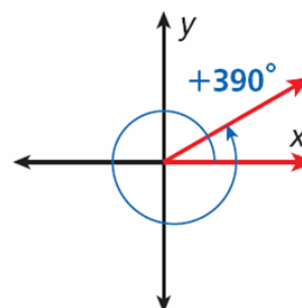
An angle is in **standard position** when its vertex is at the origin and one ray is on the positive x-axis. The **initial side** of the angle is the ray on the x-axis. The other ray is called the **terminal side** of the angle.

## 13-2 Angles of Rotation



## 13-2 Angles of Rotation

An **angle of rotation** is formed by rotating the terminal side and keeping the initial side in place. If the terminal side is rotated counterclockwise, the angle of rotation is positive. If the terminal side is rotated clockwise, the angle of rotation is negative. The terminal side can be rotated more than  $360^\circ$ .



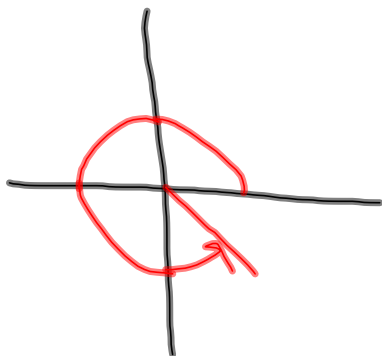
## 13-2 Angles of Rotation

### Remember!

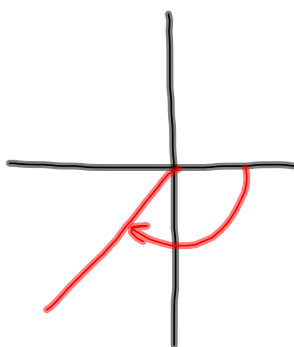
A  $360^\circ$  rotation is a complete rotation. A  $180^\circ$  rotation is one-half of a complete rotation.

**Draw an angle with the given measure in standard position.**

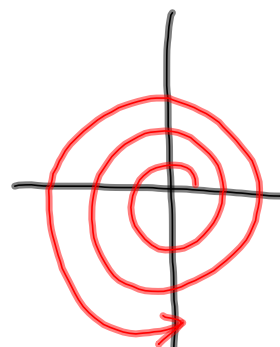
**A.**  $320^\circ$



**B.**  $-110^\circ$

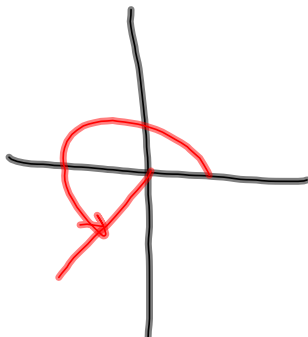


**C.**  $990^\circ$



**Draw an angle with the given measure in standard position.**

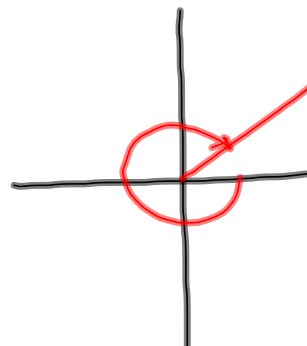
**A.**  $210^\circ$



**B.**  $1020^\circ$

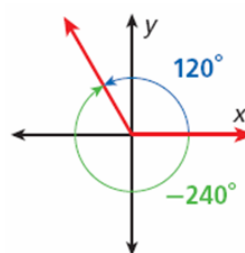


**C.**  $-300^\circ$



## 13-2 Angles of Rotation

**Coterminal angles** are angles in standard position with the same terminal side. For example, angles measuring  $120^\circ$  and  $-240^\circ$  are coterminal.



There are infinitely many coterminal angles. One way to find the measure of an angle that is coterminal with an angle  $\theta$  is to add or subtract integer multiples of  $360^\circ$ .

**Find the measures of a positive angle and a negative angle that are coterminal with each given angle.**

$$\theta = 65^\circ$$

$$65 + 360 = 425^\circ$$

$$65 - 360 = -295^\circ$$

**Find the measures of a positive angle and a negative angle that are coterminal with each given angle.**

$$\theta = 410^\circ$$

$$+ : 770^\circ$$

$$- : -310^\circ$$

**Find the measures of a positive angle and a negative angle that are coterminal with each given angle.**

$$\theta = 88^\circ$$

$$+: 88 + 360 = 448^\circ$$

$$-: 88 - 360 = -272^\circ$$

**Find the measures of a positive angle and a negative angle that are coterminal with each given angle.**

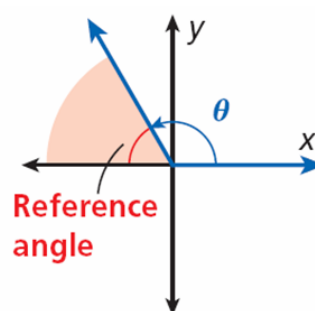
$$\theta = 500^\circ$$

**Find the measures of a positive angle and a negative angle that are coterminal with each given angle.**

$$\theta = -120^\circ$$

## 13-2 Angles of Rotation

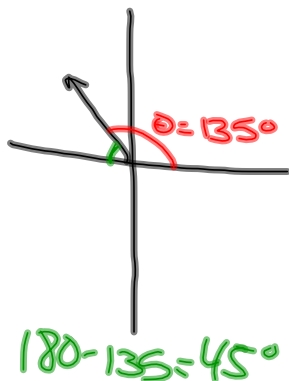
For an angle  $\theta$  in standard position, the **reference angle** is the positive acute angle formed by the terminal side of  $\theta$  and the x-axis. In Lesson 13-3, you will learn how to use reference angles to find trigonometric values of angles measuring greater than  $90^\circ$  or less than  $0^\circ$ .



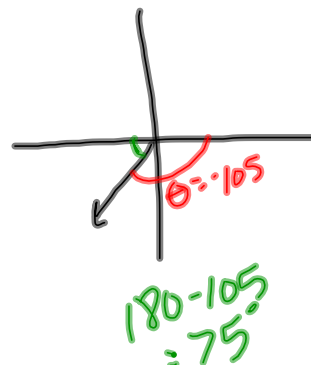


Find the measure of the reference angle for each given angle.

A.  $\theta = 135^\circ$

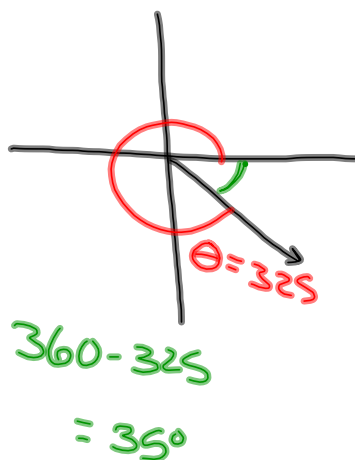


B.  $\theta = -105^\circ$



Find the measure of the reference angle for each given angle.

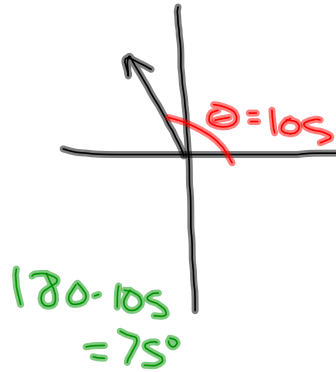
C.  $\theta = 325^\circ$



**Find the measure of the reference angle for each given angle.**

**a.  $\theta = 105^\circ$**

**b.  $\theta = -115^\circ$**

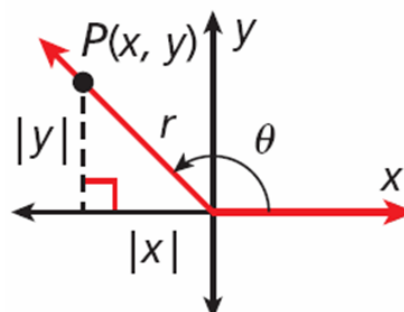


**Find the measure of the reference angle for each given angle.**

**c.  $\theta = 310^\circ$**

## 13-2 Angles of Rotation

To determine the value of the trigonometric functions for an angle  $\theta$  in standard position, begin by selecting a point  $P$  with coordinates  $(x, y)$  on the terminal side of the angle. The distance  $r$  from point  $P$  to the origin is given by  $\sqrt{x^2 + y^2}$ .



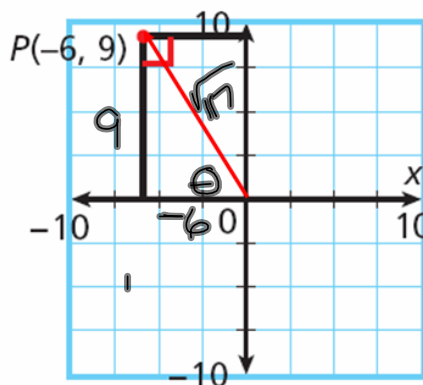
## 13-2 Angles of Rotation

### Trigonometric Functions

For a point  $P(x, y)$  on the terminal side of  $\theta$  in standard position and  $r = \sqrt{x^2 + y^2}$ ,

SINE	COSINE	TANGENT
$\sin \theta = \frac{y}{r}$	$\cos \theta = \frac{x}{r}$	$\tan \theta = \frac{y}{x}, x \neq 0$

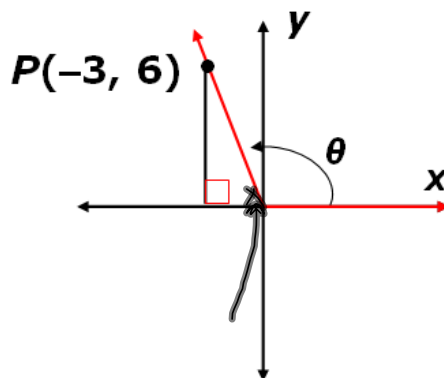
$P(-6, 9)$  is a point on the terminal side of  $\theta$  in standard position. Find the exact value of the six trigonometric functions for  $\theta$ .



$$\begin{aligned}
 r &= \sqrt{x^2 + y^2} & \sin \theta &= \frac{y}{r} \\
 r &= \sqrt{(-6)^2 + (9)^2} & \cos \theta &= \frac{x}{r} \\
 &= \sqrt{36 + 81} & \tan \theta &= \frac{y}{x} \\
 &= \sqrt{117} & \csc \theta &= \frac{r}{y} \\
 & & \sec \theta &= \frac{r}{x} \\
 & & \cot \theta &= \frac{x}{y}
 \end{aligned}$$

$P(-3, 6)$  is a point on the terminal side of  $\theta$  in standard position. Find the exact value of the six trigonometric functions for  $\theta$ .

S.O.H



$$\begin{aligned}
 r &= \sqrt{(-3)^2 + (6)^2} & \sin \theta &= \frac{y}{r} \\
 r &= \sqrt{9 + 36} & \cos \theta &= \frac{x}{r} \\
 r &= \sqrt{45} & \tan \theta &= \frac{y}{x}
 \end{aligned}$$

Homework:

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