

Warm Up

Given the measure of one of the acute angles in a right triangle, find the measure of the other acute angle.

1. 45° $180 - 90 - 45$
 $90 - 45 = 45^\circ$

2. 60° $90 - 60 = 30^\circ$

3. 24° $90 - 24 = 66^\circ$

4. 38° $90 - 38 = 52^\circ$

13-1 Right-Angle Trigonometry

Objectives

Understand and use trigonometric relationships of acute angles in triangles.

Determine side lengths of right triangles by using trigonometric functions.

13-1 Right-Angle Trigonometry

Vocabulary

trigonometric function

sine

cosine

tangent

cosecants

secant

cotangent

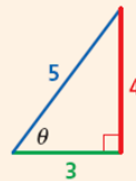
13-1 Right-Angle Trigonometry

A **trigonometric function** is a function whose rule is given by a trigonometric ratio. A *trigonometric ratio* compares the lengths of two sides of a right triangle. The Greek letter theta θ is traditionally used to represent the measure of an acute angle in a right triangle. The values of trigonometric ratios depend upon θ .

13-1 Right-Angle Trigonometry

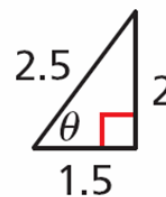
Trigonometric Functions

| WORDS | NUMBERS | SYMBOLS |
|--|-----------------------------|---|
| The sine (sin) of angle θ is the ratio of the length of the opposite leg to the length of the hypotenuse . | $\sin \theta = \frac{4}{5}$ | $\sin \theta = \frac{\text{opp.}}{\text{hyp.}}$ |
| The cosine (cos) of angle θ is the ratio of the length of the adjacent leg to the length of the hypotenuse . | $\cos \theta = \frac{3}{5}$ | $\cos \theta = \frac{\text{adj.}}{\text{hyp.}}$ |
| The tangent (tan) of angle θ is the ratio of the length of the opposite leg to the length of the adjacent leg. | $\tan \theta = \frac{4}{3}$ | $\tan \theta = \frac{\text{opp.}}{\text{adj.}}$ |



13-1 Right-Angle Trigonometry

The triangle shown at right is similar to the one in the table because their corresponding angles are congruent. No matter which triangle is used, the value of $\sin \theta$ is the same. The values of the sine and other trigonometric functions depend only on angle θ and not on the size of the triangle.



$$\sin \theta = \frac{2}{2.5} = \frac{4}{5}$$

Having trouble remembering what is sine, cosine and tangent?

Use the saying SOH-CAH-TOA

SOH: Sine, opposite/hypotenuse

CAH: Cosine, adjacent/hypotenuse

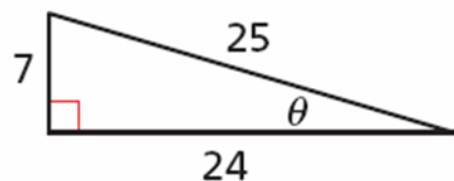
TOA: Tangent, opposite/adjacent

Find the value of the sine, cosine, and tangent functions for θ .

$$\sin \theta = \frac{7}{25}$$

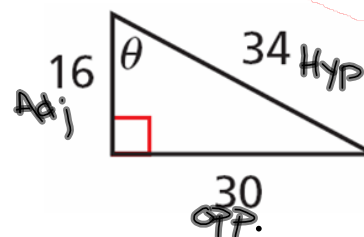
$$\cos \theta = \frac{24}{25}$$

$$\tan \theta = \frac{7}{24}$$



Find the value of the sine, cosine, and tangent functions for θ .

$$\cos \theta = \frac{16}{34}$$
$$\sin \theta = \frac{30}{34}$$
$$\tan \theta = \frac{30}{16}$$



13-1 Right-Angle Trigonometry

You will frequently need to determine the value of trigonometric ratios for 30° , 60° , and 45° angles as you solve trigonometry problems. Recall from geometry that in a 30° - 60° - 90° triangle, the ratio of the side lengths is $1:\sqrt{3}:2$, and that in a 45° - 45° - 90° triangle, the ratio of the side lengths is $1:1:\sqrt{2}$.

13-1 Right-Angle Trigonometry

| Trigonometric Ratios of Special Right Triangles | | | |
|---|---|---|--|
| Diagram | Sine | Cosine | Tangent |
| | $\sin 30^\circ = \frac{1}{2}$ $\sin 60^\circ = \frac{\sqrt{3}}{2}$ | $\cos 30^\circ = \frac{\sqrt{3}}{2}$ $\cos 60^\circ = \frac{1}{2}$ | $\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\tan 60^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}$ |
| | $\sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ | $\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ | $\tan 45^\circ = \frac{1}{1} = 1$ |

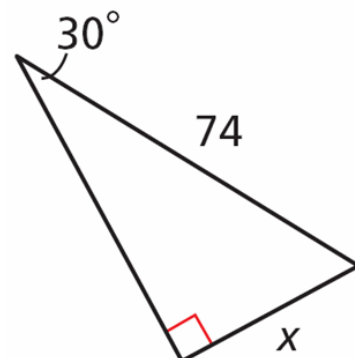
Use a trigonometric function to find the value of x .

$$74 \cdot \sin 30 = \frac{x}{74} \quad 74$$

$$x = 37$$

$$\frac{1}{2} = \frac{x}{74}$$

$$x = 37$$



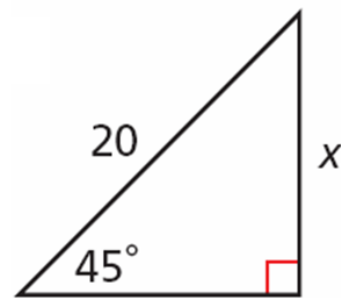
Use a trigonometric function to find the value of x .

$$\sin 45 = \frac{x}{20}$$

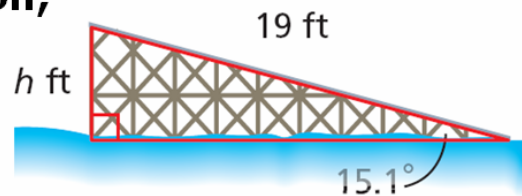
$$\frac{\sqrt{2}}{2} = \frac{x}{20}$$

$$\frac{20\sqrt{2}}{2} = x$$

$$10\sqrt{2} = x$$



In a waterskiing competition, a jump ramp has the measurements shown. To the nearest foot, what is the height h above water that a skier leaves the ramp?



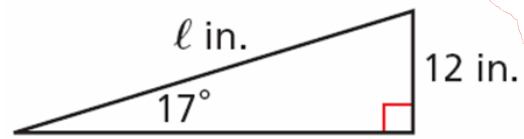
$$\sin 15.1 = \frac{h}{19}$$

$$h = 19 \cdot \sin 15.1$$

$$= \cancel{23} \text{ ft}$$

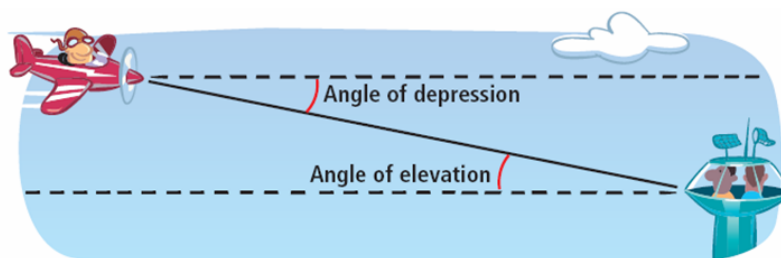
$$4.9$$

A skateboard ramp will have a height of 12 in., and the angle between the ramp and the ground will be 17° . To the nearest inch, what will be the length l of the ramp?

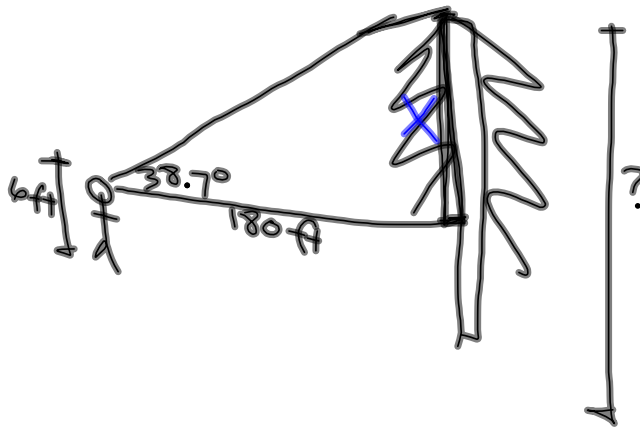


13-1 Right-Angle Trigonometry

When an object is above or below another object, you can find distances indirectly by using the *angle of elevation* or the *angle of depression* between the objects.



A biologist whose eye level is 6 ft above the ground measures the angle of elevation to the top of a tree to be 38.7° . If the biologist is standing 180 ft from the tree's base, what is the height of the tree to the nearest foot?



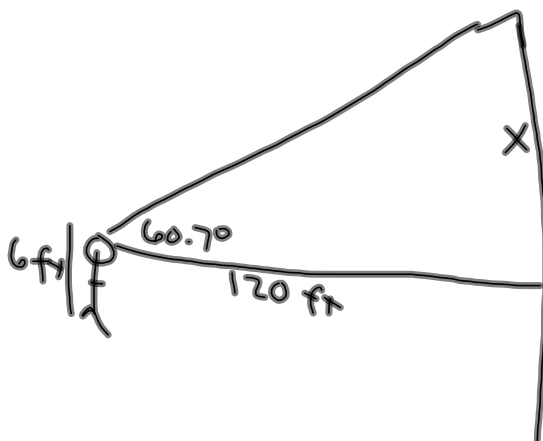
$$180 \tan 38.7 = \frac{x}{180} \cdot 180$$

$$x = 144.2 \text{ ft}$$

$$144 + 6$$

$$= 150.2 \text{ ft}$$

A surveyor whose eye level is 6 ft above the ground measures the angle of elevation to the top of the highest hill on a roller coaster to be 60.7° . If the surveyor is standing 120 ft from the hill's base, what is the height of the hill to the nearest foot?



$$\tan 60.7 = \frac{x}{120}$$

$$x = 213.8$$

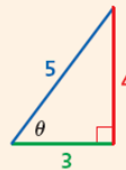
$$219.8 \text{ ft}$$

13-1 Right-Angle Trigonometry

The reciprocals of the sine, cosine, and tangent ratios are also trigonometric ratios. They are trigonometric functions, *cosecant*, *secant*, and *cotangent*.

Reciprocal Trigonometric Functions

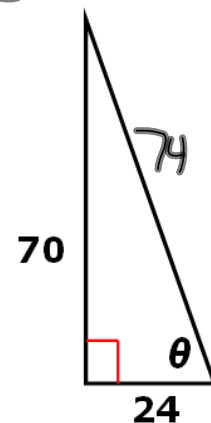
| WORDS | NUMBERS | SYMBOLS |
|---|------------------------------------|--|
| The cosecant (csc) of angle θ is the reciprocal of the sine function. | $\text{csc } \theta = \frac{5}{4}$ | $\text{csc } \theta = \frac{1}{\sin \theta} = \frac{\text{hyp.}}{\text{opp.}}$ |
| The secant (sec) of angle θ is the reciprocal of the cosine function. | $\text{sec } \theta = \frac{5}{3}$ | $\text{sec } \theta = \frac{1}{\cos \theta} = \frac{\text{hyp.}}{\text{adj.}}$ |
| The cotangent (cot) of angle θ is the reciprocal of the tangent function. | $\text{cot } \theta = \frac{3}{4}$ | $\text{cot } \theta = \frac{1}{\tan \theta} = \frac{\text{adj.}}{\text{opp.}}$ |



Find the values of the six trigonometric functions for θ .

$$\begin{aligned} \cos \theta &= \frac{24}{74} & \sec \theta &= \frac{74}{24} \\ \sin \theta &= \frac{70}{74} & \csc \theta &= \frac{74}{70} \\ \tan \theta &= \frac{70}{24} & \cot \theta &= \frac{24}{70} \end{aligned}$$

$$24^2 + 70^2 = c^2$$



13-1 Right-Angle Trigonometry

Helpful Hint

In each reciprocal pair of trigonometric functions, there is exactly one "co"

$$\text{cosecant } \theta = \frac{1}{\text{sine } \theta}$$

$$\text{secant } \theta = \frac{1}{\text{cosine } \theta}$$

$$\text{cotangent } \theta = \frac{1}{\text{tangent } \theta}$$

Find the values of the six trigonometric functions for θ .

$$\sin \theta = \frac{80}{82}$$

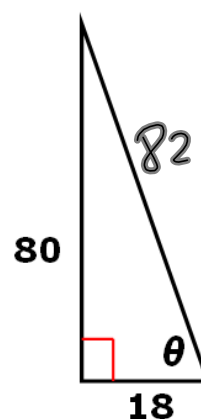
$$\csc \theta = \frac{82}{80}$$

$$\cos \theta = \frac{18}{82}$$

$$\sec \theta = \frac{82}{18}$$

$$\tan \theta = \frac{80}{18}$$

$$\cot \theta = \frac{18}{80}$$



Homework:

p.933 #13-19, 21-24, 27-28, 30-32