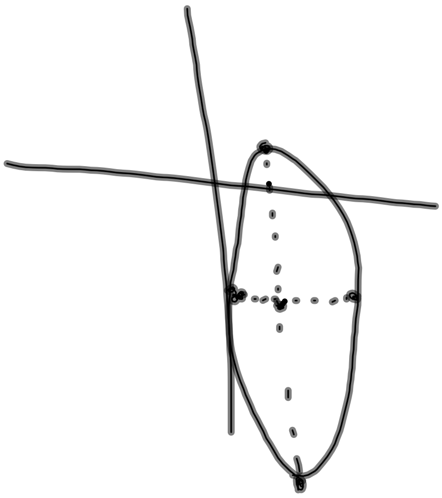
Warm Up:

Graph the ellipse $\frac{(x-2)^2}{4} + \frac{(y+5)^2}{36} = 1$.

(2,-5)



10-4 Hyperbolas

Objectives

Write the standard equation for a hyperbola.

Graph a hyperbola, and identify its center, vertices, co-vertices, foci, and asymptotes.

10-4 Hyperbolas

Vocabulary

hyperbola
focus of a hyperbola
branch of a hyperbola
transverse axis
vertices of a hyperbola
conjugate axis
co-vertices of a hyperbola

10-4 Hyperbolas

What would happen if you pulled the two foci of an ellipse so far apart that they moved outside the ellipse? The result would be a *hyperbola*, another conic section.

A **hyperbola** is a set of points $P(x, y)$ in a plane such that the difference of the distances from P to fixed points F_1 and F_2, the **foci**, is constant. For a hyperbola, $d = |PF_1 - PF_2|$, where d is the constant difference. You can use the distance formula to find the equation of a hyperbola.

Holt Algebra 2
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Just like an ellipse we are going to use constant lengths, this one just happens to be a constant difference.

$d=|PF_1-PF_2|$

Find the constant difference for a hyperbola with foci F_1 (–8, 0) and F_2 (8, 0) and the point on the hyperbola (8, 30).

$d = PF_1 - PF_2$

$= \sqrt{(8--8)^2+(30-0)^2} - \sqrt{(8-8)^2+(30-0)^2}$

$= \sqrt{16^2+30^2} - \sqrt{30^2}$

$= \sqrt{1124} - \sqrt{900}$

$= 34-30$

$= 4$

Find the constant difference for a hyperbola with foci F_1 (0, –10) and F_2 (0, 10) and the point on the hyperbola (6, 7.5).

$d = PF_1 - PF_2$

$\sqrt{(6-0)^2+(7.5-{-10})^2} - \sqrt{(6-0)^2+(7.5-10)^2}$

$\sqrt{6^2+(17.5)^2} - \sqrt{6^2+(-2.5)^2}$

$18.5 - 6.5$

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10-4 Hyperbolas

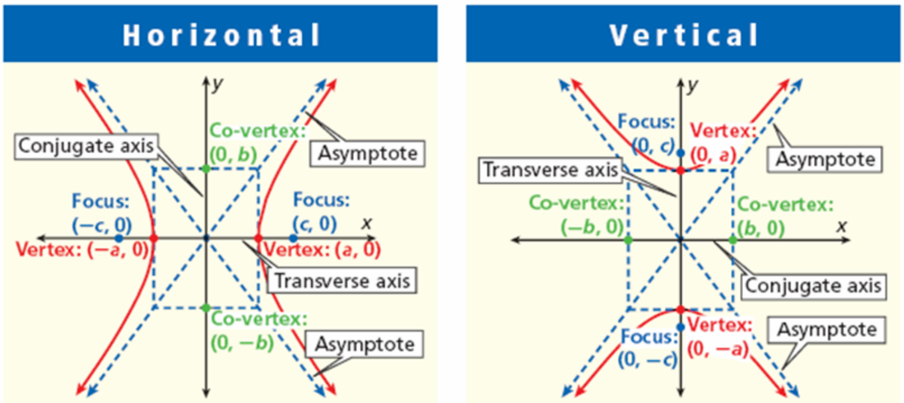
As the graphs in the following table show, a hyperbola contains two symmetrical parts called **branches**.

A hyperbola also has two axes of symmetry. The **transverse axis** of symmetry contains the vertices and, if it were extended, the foci of the hyperbola. The **vertices of a hyperbola** are the endpoints of the transverse axis.

The **conjugate axis** of symmetry separates the two branches of the hyperbola. The **co-vertices of a hyperbola** are the endpoints of the conjugate axis. The transverse axis is not always longer than the conjugate axis.

10-4 Hyperbolas

The standard form of the equation of a hyperbola depends on whether the hyperbola's transverse axis is horizontal or vertical.



10-4 Hyperbolas

The values a, b, and c, are related by the equation $c^2 = a^2 + b^2$. Also note that the length of the transverse axis is $2a$ and the length of the conjugate is $2b$.

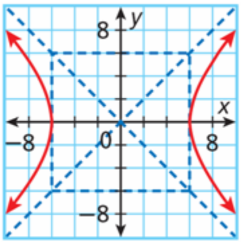
Standard Form for the Equation of a Hyperbola Center at (0, 0)

TRANSVERSE AXIS	HORIZONTAL	VERTICAL
Equation	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
Vertices	$(a, 0), (-a, 0)$	$(0, a), (0, -a)$
Foci	$(c, 0), (-c, 0)$	$(0, c), (0, -c)$
Co-vertices	$(0, b), (0, -b)$	$(b, 0), (-b, 0)$
Asymptotes	$y = \pm\frac{b}{a}x$	$y = \pm\frac{a}{b}x$

To write equations identify the a, b and c values. If you are missing a or b then use the Pythagorean Theorem to find them.

Then identify if you have a horizontal or vertical hyperbola and choose the right equation.

Write an equation in standard form for each hyperbola.



Write an equation in standard form for each hyperbola.

The hyperbola with center at the origin, vertex (4, 0), and focus (10, 0).

x

Center: (0,0)

$a=4$

$c=10$

$b=\sqrt{84}$

$a^2+b^2=c^2$

$16+b^2=100$

$b^2=84$

$b=\sqrt{84}$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$$\frac{x^2}{16}-\frac{y^2}{84}=1$$

Write an equation in standard form for each hyperbola.

Vertex (0, 9), co-vertex (7, 0)

y

$a=9$

$b=7$

$$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$$

$$\frac{y^2}{81}-\frac{x^2}{49}=1$$

Write an equation in standard form for each hyperbola.

Vertex (8, 0), focus (10, 0)

$a=8$

$c=10$

$b=6$

$c^2=a^2+b^2$

$100=64+b^2$

$b^2=36$

$b=6$

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\frac{x^2}{64}-\frac{y^2}{36}=1$

10-4 Hyperbolas

As with circles and ellipses, hyperbolas do not have to be centered at the origin.

Standard Form for the Equation of a Hyperbola Center at (h, k)

TRANSVERSE AXIS	HORIZONTAL	VERTICAL
Equation	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$
Vertices	$(h+a, k), (h-a, k)$	$(h, k+a), (h, k-a)$
Foci	$(h+c, k), (h-c, k)$	$(h, k+c), (h, k-c)$
Co-vertices	$(h, k+b), (h, k-b)$	$(h+b, k), (h-b, k)$
Asymptotes	$y-k=\pm\frac{b}{a}(x-h)$	$y-k=\pm\frac{a}{b}(x-h)$

Find the vertices, co-vertices, and asymptotes of each hyperbola, and then graph.

$$\frac{x^2}{49} - \frac{y^2}{9} = 1$$

$a=7$
$b=3$

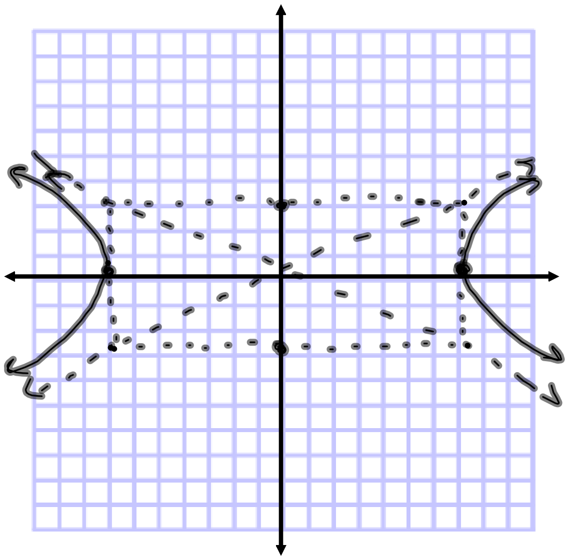
Horz.

Vert: (7,0) (-7,0)

Co-Vert: (0,3) (0,-3)

asy: $y = \pm \frac{b}{a}x$

$y = \frac{3}{7}x \quad y = -\frac{3}{7}x$



Find the vertices, co-vertices, and asymptotes of each hyperbola, and then graph.

$$\frac{(x-3)^2}{9} - \frac{(y+5)^2}{49} = 1$$

Horz.

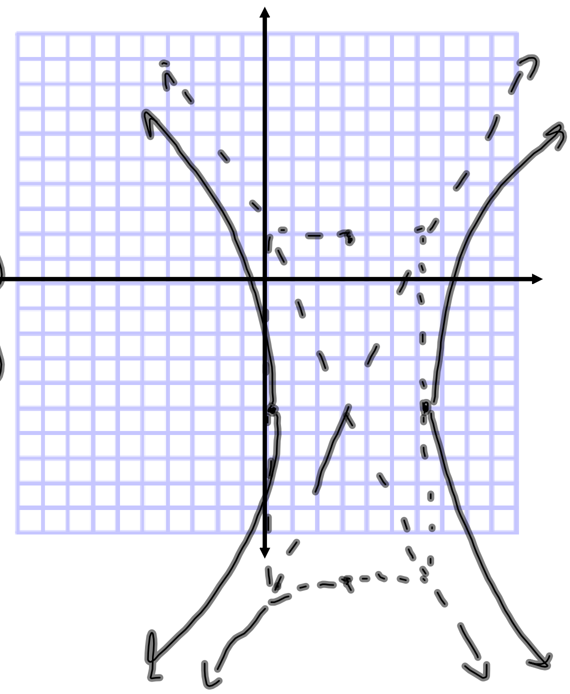
(3,-5)

$h=3$
$k=-5$
$a=3$
$b=7$

Vert: (6,-5) (0,-5)

Co-Vert: (3,2) (3,-12)

$y+5 = \pm\frac{7}{3}(x-3)$



Find the vertices, co-vertices, and asymptotes of each hyperbola, and then graph.

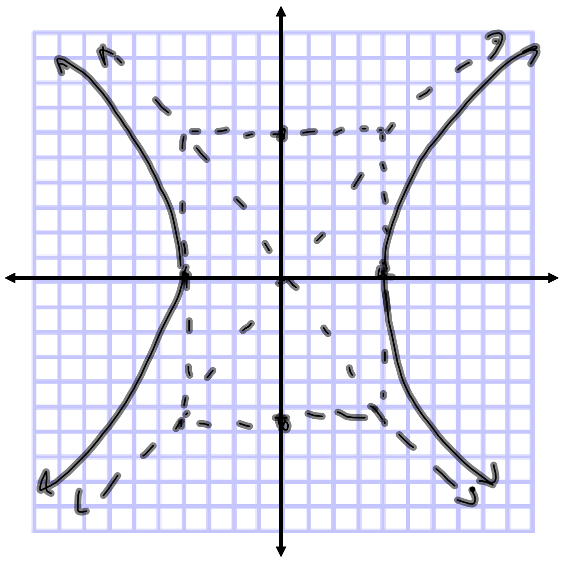
$$\frac{x^2}{16} - \frac{y^2}{36} = 1$$

Horz. $a=4$ $b=6$

Vert: (4,0) (-4,0)

Co-Vert: (0,6) (0,-6)

Asy: $y = \pm \frac{6}{4} x$



Homework:

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