

Warm Up:

Think of any questions you may have from 8-1 through 8-5.

Solve $\frac{4}{x-3} \geq 2$

$$\begin{array}{l} \underline{+} \\ 4 \geq 2x - 6 \\ 10 \geq 2x \\ 5 \geq x \end{array}$$

$$\begin{array}{l} \underline{-} \\ 4 \leq 2x - 6 \\ 10 \leq 2x \\ 5 \leq x \\ \downarrow \\ 3 \leq x \end{array}$$

8-1: Variations

There are 4 types of variations:

-Direct variation: $y = kx$

-Joint variation: $y = kxz$

-Inverse variation: $y = \frac{k}{x}$

-Combined variation: $y = \frac{kx}{z}$

To solve these problems find k and then solve for the missing variable.

The perimeter P of a regular dodecagon varies directly as the side length s , and $P = 18$ in. when $s = 1.5$ in. Find s when $P = 75$ in.

$$P = ks$$

$$18 = k \cdot 1.5$$

$$k = 12$$

$$75 = 12 \cdot s$$

$$s = 6.2$$

The time t needed to complete a certain race varies inversely as the runner's average speed s . If a runner with an average speed of 8.82 mi/h completes the race in 2.97 h, what is the average speed of a runner who completes the race in 3.5 h?

$$t = \frac{k}{s}$$

$$2.97 = \frac{k}{8.82}$$

$$k = 26.2$$

$$3.5 = \frac{26.2}{s}$$

$$s = \frac{26.2}{3.5}$$

The volume V of a gas varies inversely as the pressure P and directly as the temperature T . A certain gas has a volume of 10 liters (L), a temperature of 300 kelvins (K), and a pressure of 1.5 atmospheres (atm). If the gas is heated to 400K, and has a pressure of 1 atm, what is its volume?

$$V = \frac{kT}{P}$$

8-2: Multiplying and Dividing Rational Expressions

To solve these problems factor, reduce, multiply/divide, reduce again.

*Remember when dividing you need to flip and multiply before cross-reducing.

$$\frac{x-3}{4x+20} \cdot \frac{x+5}{x^2-9}$$

$$\frac{\cancel{x-3}}{4\cancel{(x+5)}} \cdot \frac{\cancel{(x+5)}}{\cancel{(x-3)}(x+3)}$$

$$\frac{1}{4(x+3)}$$

$$\frac{x^4 - 9x^2}{x^2 - 4x + 3} \div \frac{x^4 + 2x^3 - 8x^2}{x^2 - 16}$$

$$\frac{x^4 - 9x^2}{x^2 - 4x + 3} \cdot \frac{x^2 - 16}{x^4 + 2x^3 - 8x^2}$$

$$\frac{\cancel{x^2}(\cancel{x-3})(x+3)}{(\cancel{x-3})(x-1)} \cdot \frac{(x-4)\cancel{(x+4)}}{\cancel{x^2}(\cancel{x+4})(x-2)}$$

$$\frac{(x+3)(x-4)}{(x-1)(x-2)}$$

8-3: Adding/Subtracting Rational Expressions

First thing is find a LCM. To do this get each expression into its most factored piece ie: prime numbers and variables.

Ex: Find the LCM of $12x^5y^2$ and $21x^2y^7$

$$\cancel{2} \cdot \cancel{2} \cdot 3 \cdot x^5 \cdot y^2 \quad \cancel{3} \cdot 7 \cdot x^2 \cdot y^7$$

$$2 \cdot 2 \cdot 3 \cdot 7 \cdot x^5 \cdot y^7$$

$$84x^5y^7$$

To add or subtract you need a common denominator. Find the LCM to get it and then multiply the numerator and denominator to get to the common denominator.

$$\frac{x-3}{x^2+3x-4} + \frac{2x}{x+4}$$

$$\frac{x-3}{(x+4)(x-1)} + \frac{2x(x-1)}{(x+4)(x-1)}$$

$$\frac{x-3}{(x+4)(x-1)} + \frac{2x^2-2x}{(x+4)(x-1)}$$

$$\frac{2x^2-x-3}{(x+4)(x-1)}$$

$$\frac{2x^2 + 64}{x^2 - 64} - \frac{x - 4}{x + 8}$$

$$\frac{2x^2 + 64}{(x-8)(x+8)} - \frac{x-4}{x+8} \frac{(x-8)}{(x-8)}$$

$$\frac{2x^2 + 64}{(x-8)(x+8)} - \frac{x^2 - 12x + 32}{(x-8)(x+8)}$$

$$\frac{x^2 + 12x + 32}{(x-8)(x+8)}$$

When you have a fraction over a fraction you have a complex fraction. To simplify just write as a division problem and simplify.

$$\frac{\frac{x+1}{x^2-1}}{\frac{x}{x-1}}$$

$$\frac{x+1}{x^2-1} \div \frac{x}{x-1}$$

$$\frac{x+1}{(x+1)(x-1)} \cdot \frac{x-1}{x}$$

$$\frac{1}{x}$$

8-4: Rational Functions

The first thing is transforming functions. This works the exact same way as every other function. Add to the end to move up/down and add to x to move left/right.

To find vertical asymptotes: Set denominator equal to zero and solve for x.

To find zeros: Set numerator to zero and solve for x.

To find horizontal asymptotes:

-If numerator has higher degree than the denominator there is no horiz. asymptote.

-If denominator has higher degree than the numerator there is a horiz. asymptote at $y=0$.

If numerator and denominator have same degree. Then horiz. asymptote occurs at

$$y = \frac{\text{leading coefficient of } p}{\text{leading coefficient of } q}$$

Identify the zeros and asymptotes of the function. Then graph.

$$f(x) = \frac{x^2 - 3x - 4}{x}$$

V.A.

$$x=0$$

H.A.

None

0

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$$x=4 \text{ and } -1$$

8-5: Solving rational equalities

Multiply the entire function by an expression that removes all fractions. This is found by multiplying all the denominators together.

Remember to always check for extraneous solutions.

$$\left(\frac{6}{x} + \frac{5}{4} = -\frac{7}{4} \right) 4x$$

$$24 + 5x = -7x$$

Sometimes you may have an inequality.
Remember to check both the positive
AND the negative case and adjust your
solutions accordingly.

$$\frac{9}{x+3} < 6$$

$$\begin{array}{c} \pm \\ 9 < 6x + 18 \end{array}$$

$$-\frac{3}{2} < x \checkmark$$

$$\begin{array}{c} - \\ 9 > 6x + 18 \end{array}$$

$$-\frac{3}{2} > x$$

$$-3 > x$$

Any other questions or
problems you would like
to try yourself?

When you are done write your partner's name in the space on the back and let me know how well you worked as a group.