Warm Up:

An online computer game company has 10,000 subscribers paying $8 per month. Their research shows that for every 25-cent reduction in their fee, they will attract another 500 users. Use a table and an equation to find the fee that the company should charge to maximize their revenue.

Hint: Find quadratic regression with fee as independent variable and revenue as dependent variable.

9-2 Piecewise Functions

Objectives

Write and graph piecewise functions.

Use piecewise functions to describe real-world situations.

9-2 Piecewise Functions

Vocabulary

piecewise function
step function

9-2 Piecewise Functions

A **piecewise function** is a function that is a combination of one or more functions. The rule for a piecewise function is different for different parts, or pieces, of the domain. For instance, movie ticket prices are often different for different age groups. So the function for movie ticket prices would assign a different value (ticket price) for each domain interval (age group).

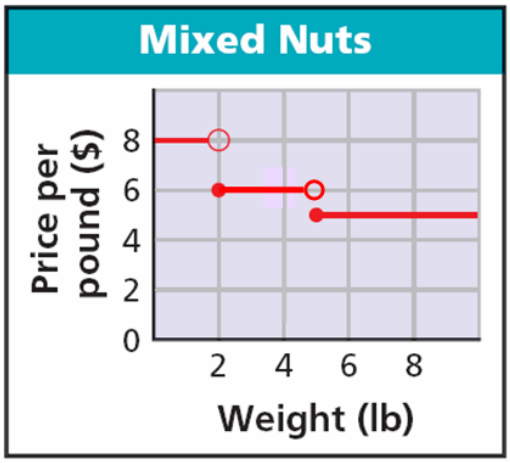
9-2 Piecewise Functions

Remember!

When using interval notation, square brackets [ ] indicate an included endpoint, and parentheses ( ) indicate an excluded endpoint. (Lesson 1-1)

Create a table and a verbal description to represent the graph.

$$f(x) = \begin{cases} 8, & x < 2 \\ 6, & 2 \le x < 5 \\ 5, & x \ge 5 \end{cases}$$



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Create a table and a verbal description to represent the graph.

$$f(x) = \begin{cases} 27, & x < 12 \\ 24, & 12 \le x < 16 \\ 12, & x \ge 16 \end{cases}$$

9-2 Piecewise Functions

A piecewise function that is constant for each interval of its domain, such as the ticket price function, is called a **step function**. You can describe piecewise functions with a function rule. The rule for the movie ticket prices from Example 1 on page 662 is shown.

$$f(x) = \begin{cases} 5 & \text{if } 0 < x < 13 \\ 9 & \text{if } 13 \le x < 55 \\ 6.5 & \text{if } x \ge 55 \end{cases}$$

9-2 Piecewise Functions

Read this as "*f* of *x* is 5 if *x* is greater than 0 and less than 13, 9 if *x* is greater than or equal to 13 and less than 55, and 6.5 if *x* is greater than or equal to 55."

$$f(x) = \begin{cases} 5 & \text{if } 0 < x < 13 \\ 9 & \text{if } 13 \le x < 55 \\ 6.5 & \text{if } x \ge 55 \end{cases}$$

To evaluate any piecewise function for a specific input, find the interval of the domain that contains that input and then use the rule for that interval.

Evaluate each piecewise function for $x = -1$ and $x = 4$.

$$h(x) = \begin{cases} 2x + 1 & \text{if } x \le 2 \\ x^2 - 4 & \text{if } x > 2 \end{cases}$$

$2(-1)+1$
$-2+1$
$h(-1) = -1$

$4^2 - 4$
$16 - 4$
$h(4) = 12$

Evaluate each piecewise function for $x = -1$ and $x = 4$.

$$g(x) = \begin{cases} 2^x & \text{if } x \le -1 \\ 5x & \text{if } x > -1 \end{cases}$$

2^{-1}

$g(-1) = \frac{1}{2}$

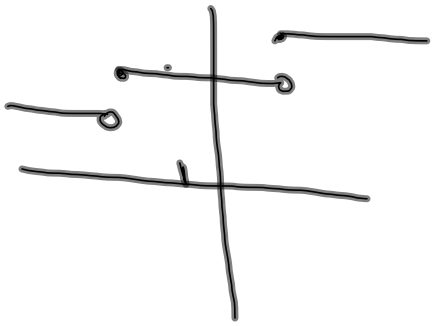
$5(4)$
$g(4) = 20$

Evaluate each piecewise function for $x = -1$ and $x = 3$.

$$f(x) = \begin{cases} 12 & \text{if } x < -3 \\ 15 & \text{if } -3 \le x < 6 \\ 20 & \text{if } x \ge 6 \end{cases}$$

$g(-1) = 15$

$g(3) = 15$



Evaluate each piecewise function for $x = -1$ and $x = 3$.

$$g(x) = \begin{cases} 3x^2 + 1 & \text{if } x < 0 \\ 5x - 2 & \text{if } x \ge 0 \end{cases}$$

9-2 Piecewise Functions

You can graph a piecewise function by graphing each piece of the function.

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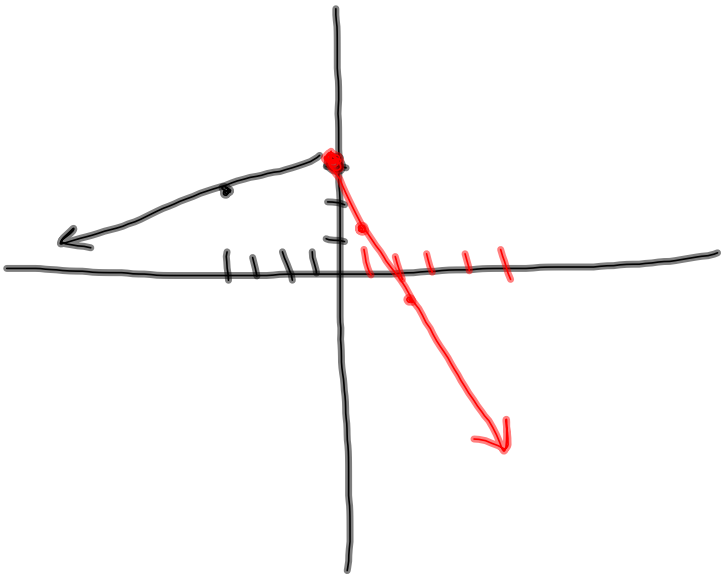
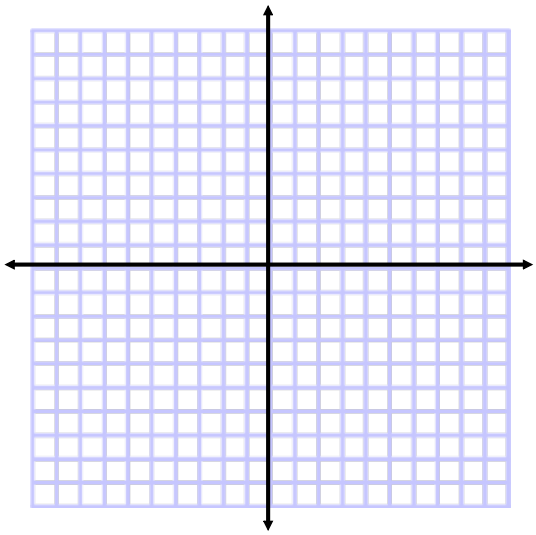
Graph each function.

$$g(x) = \begin{cases} \frac{1}{4}x + 3 & \text{if } x < 0 \\ -2x + 3 & \text{if } x \geq 0 \end{cases}$$



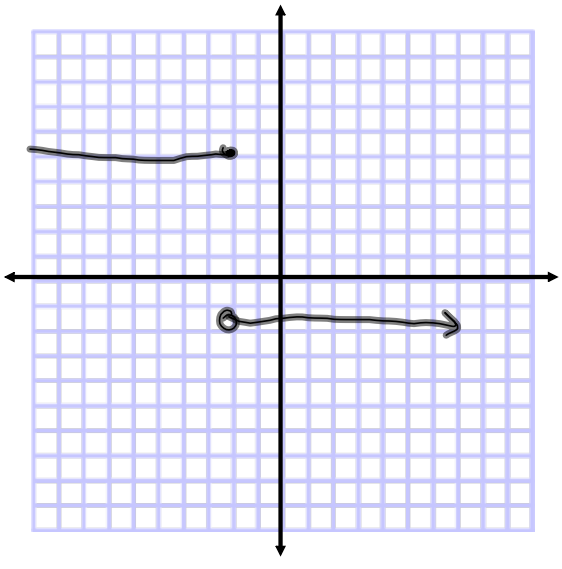
Graph each function.

$$g(x) = \begin{cases} x^2 - 3 & \text{if } x < 0 \\ \frac{1}{2}x - 3 & \text{if } 0 \le x < 4 \\ (x-4)^2 - 1 & \text{if } x \ge 4 \end{cases}$$



Graph the function.

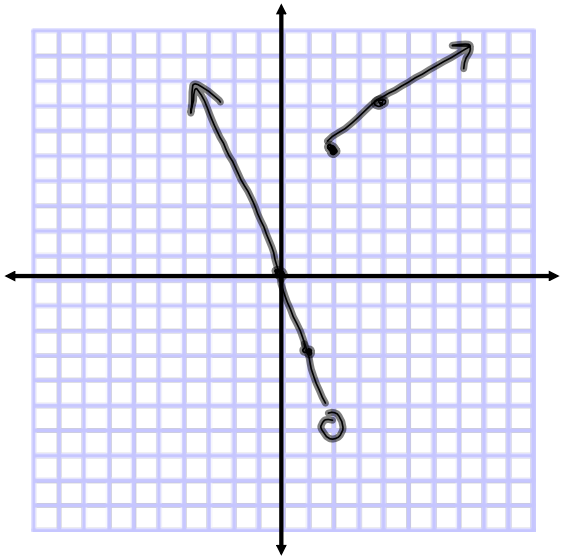
$$f(x) = \begin{cases} 4 & \text{if } x \le -1 \\ -2 & \text{if } x > -1 \end{cases}$$



Graph the function.

$$g(x) = \begin{cases} -3x & \text{if } x < 2 \\ x + 3 & \text{if } x \geq 2 \end{cases}$$

x	y
2	5
4	7



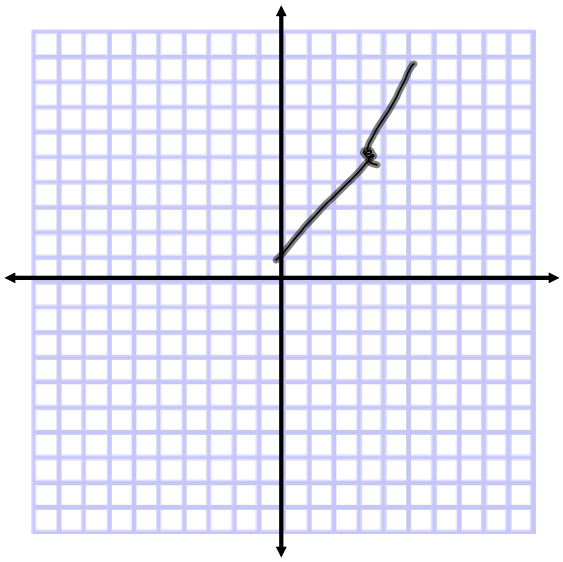
9-2 Piecewise Functions

Notice that piecewise functions are not necessarily *continuous*, meaning that the graph of the function may have breaks or gaps.

To write the rule for a piecewise function, determine where the domain is divided and write a separate rule for each piece. Combine the pieces by using the correct notation.

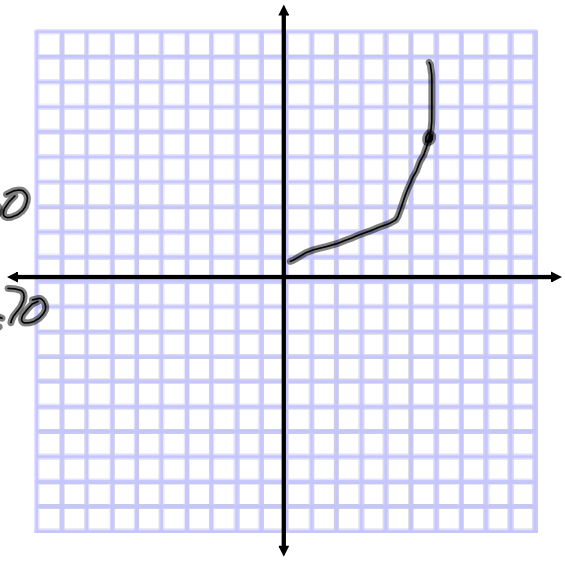
Shelly earns $8 an hour. She earns $12 an hour for each hour over 40 that she works. Sketch a graph of Shelly's earnings versus the number of hours that she works up to 60 hours. Then write a piecewise function for the graph.

$$f(x) = \begin{cases} 8x, & 0 \le x \le 40 \\ 12x + 320, & 40 < x \le 60 \end{cases}$$



Write and graph a piecewise function for the following situation. A house painter charges $12 per hour for the first 40 hours he works, time and a half for the 10 hours after that, and double time for all hours after that. How much does he earn for a 70-hour week?

$$f(x) \begin{cases} 12x, & 0 \le x \le 40 \\ 18x + 480, & 40 < x \le 50 \\ 24x + 660, & 50 < x \le 70 \end{cases}$$



Homework:

p. 666 #9-14, 16-19, 21-23, 30, 33-35