Warm Up:

Add. Identify any *x*-values for which the expression is undefined.

$$\frac{x}{x+3} + \frac{2x+6}{x^2+6x+9}$$

$$\frac{x}{x+3} + \frac{2x+6}{(x+3)(x+3)}$$

$$\frac{x(x+3)}{(x+3)^2} + \frac{2x+6}{(x+3)^2}$$

$$\frac{x^2+5x+6}{(x+3)^2} = \frac{(x+2)(x+3)}{(x+3)^2}$$

$$\frac{x+2}{x+3}$$

*Get out a half sheet of paper.

8-4 Rational Functions

Objectives

Graph rational functions.

Transform rational functions by changing parameters.

8-4 Rational Functions

Vocabulary

rational function
discontinuous function
continuous function
hole (in a graph)

8-4 Rational Functions

A **rational function** is a function whose rule can be written as a ratio of two polynomials. The parent rational function is $f(x) = \frac{1}{x}$. Its graph is a *hyperbola*, which has two separate branches. You will learn more about hyperbolas in Chapter 10.

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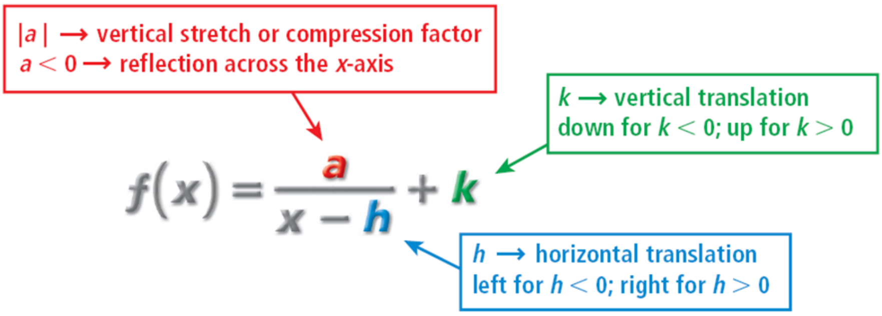
8-4 Rational Functions

The rational function $f(x) = \frac{1}{x}$ can be transformed by using methods similar to those used to transform other types of functions.



Transforming a rational function is the same as transforming any other function. Pay attention to where things are multiplied and added and follow the same rules you have been using all year.

Using the graph of $f(x) = \frac{1}{x}$ as a guide, describe the transformation and graph each function.

A. $g(x) = \frac{1}{x+2}$

Horz. slide
left 2

B. $g(x) = \frac{1}{x} - 3$

Vert.
down 3

Using the graph of $f(x) = \frac{1}{x}$ as a guide, describe the transformation and graph each function.

a. $g(x) = \frac{1}{x+4}$

Horz.
left 4

b. $g(x) = \frac{1}{x} + 1$

Up 1

The values of h and k affect the locations of the asymptotes, the domain, and the range of rational functions whose graphs are hyperbolas.

Rational Functions

For a rational function of the form $f(x) = \frac{a}{x - h} + k$,

- the graph is a hyperbola.
- there is a vertical asymptote at the line $x = h$, and the domain is $\{x \mid x \neq h\}$.
- there is a horizontal asymptote at the line $y = k$, and the range is $\{y \mid y \neq k\}$.

Just remember that:

-Vertical asymptote occurs at
x = # added to x value

-Horizontal asymptote occurs at
y = # added to end of function

Identify the asymptotes, domain, and range of the function $g(x) = \dfrac{1}{x + 3} - 2$.

left 3 $x = -3$

down 2 $y = -2$

D: Reals, $x \neq -3$

R: Reals, $y \neq -2$

Identify the asymptotes, domain, and range of the function $g(x) = \dfrac{1}{x - 3} - 5$.

Right 3 $x = 3$

Down 5 $y = -5$

D: Reals, $x \neq 3$

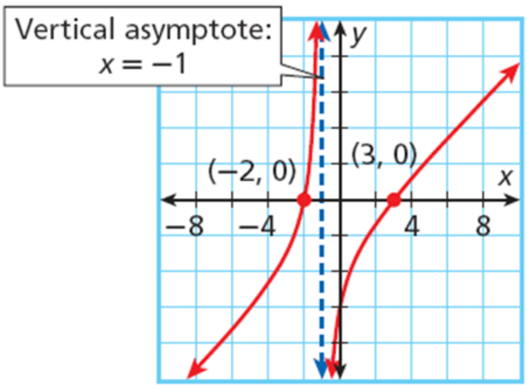
R: Reals, $y \neq -5$

A **discontinuous function** is a function whose graph has one or more gaps or breaks. The hyperbola graphed in Example 2 and many other rational functions are discontinuous functions.

A **continuous function** is a function whose graph has no gaps or breaks. The functions you have studied before this, including linear, quadratic, polynomial, exponential, and logarithmic functions, are continuous functions.

The graphs of some rational functions are not hyperbolas. Consider the rational function $f(x) = \frac{(x-3)(x+2)}{x+1}$ and its graph.

The numerator of this function is 0 when $x = 3$ or $x = -2$. Therefore, the function has x-intercepts at -2 and 3. The denominator of this function is 0 when $x = -1$. As a result, the graph of the function has a vertical asymptote at the line $x = -1$.



Zeros and Vertical Asymptotes Rational Functions

If $f(x) = \frac{p(x)}{q(x)}$, where p and q are polynomial functions in standard form with no common factors other than 1, then the function f has

- zeros at each real value of x for which $p(x) = 0$.
- a vertical asymptote at each real value of x for which $q(x) = 0$.

To find the zeroes:
Set the numerator equal to 0 and solve for x.
Those are your zeroes. Why?

To find vertical asymptotes:
Set denominator equal to 0 and solve for x.
Those are your vertical asymptotes. Why?

Identify the zeros and vertical asymptotes of $f(x) = \frac{(x^2 + 3x - 4)}{x + 3}$.

Zeros:

$x^2+3x-4=0$

$(x+4)(x-1)=0$

$x=-4$ and 1

Asy:

$x+3=0$

$x=-3$

Identify the zeros and vertical asymptotes of $f(x) = \frac{(x^2 + 7x + 6)}{x + 3}$.

Zeros:

$x^2+7x+6=0$

$(x+6)(x+1)$

$x=-6$ and -1

Asy.

$x+3=0$

$x=-3$

Some rational functions, including those whose graphs are hyperbolas, have a horizontal asymptote. The existence and location of a horizontal asymptote depends on the degrees of the polynomials that make up the rational function.

Note that the graph of a rational function can sometimes cross a horizontal asymptote. However, the graph will approach the asymptote when $|x|$ is large.

Horizontal Asymptotes **Rational Functions**

Let $f(x) = \frac{p(x)}{q(x)}$, where p and q are polynomial functions in standard form with no common factors other than 1. The graph of f has at most one horizontal asymptote.

- If degree of $p >$ degree of q, there is no horizontal asymptote.
- If degree of $p <$ degree of q, the horizontal asymptote is the line $y = 0$.
- If degree of $p =$ degree of q, the horizontal asymptote is the line

$$y = \frac{\text{leading coefficient of } p}{\text{leading coefficient of } q}.$$

-If numerator has higher degree than the denominator there is no horiz. asymptote.

-If denominator has higher degree than the numerator there is a horiz. asymptote at y=0.

If numerator and denominator have same degree. Then horiz. asypmtote occurs at

$$y = \frac{\text{leading coefficient of } p}{\text{leading coefficient of } q}.$$

Identify the zeros and asymptotes of the function. Then graph.

$$f(x) = \frac{x^2 - 3x - 4}{x}$$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4$ and -1

V.A.

$x = 0$

No H.A.

Identify the zeros and asymptotes of the function. Then graph.

$$f(x) = \frac{x - 2}{x^2 - 1}$$

Zero:
$x - 2 = 0$
$x = 2$

V.A
$x^2 - 1 = 0$
$(x-1)(x+1) = 0$
$x = 1$ and -1

H.A.
$y = 0$

Identify the zeros and asymptotes of the function. Then graph.

$$f(x) = \frac{4x - 12}{x - 1}$$

Zero:
$4x - 12 = 0$
$4x = 12$
$x = 3$

V.A.
$x - 1 = 0$
$x = 1$

H.A.
$y = \frac{4}{1} = y = 4$

Identify the zeros and asymptotes of the function. Then graph.

$$f(x) = \frac{x^2 + 2x - 15}{x - 1}$$

In some cases, both the numerator and the denominator of a rational function will equal 0 for a particular value of x. As a result, the function will be undefined at this x-value. If this is the case, the graph of the function may have a *hole*. A **hole** is an omitted point in a graph.

Holes in Graphs | **Rational Functions**

If a rational function has the same factor $x - b$ in both the numerator and the denominator, then there is a hole in the graph at the point where $x = b$, unless the line $x = b$ is a vertical asymptote.

Identify holes in the graph of $f(x) = \frac{x^2 - 9}{x - 3}$. Then graph.

$$\frac{(x-3)(x+3)}{(x-3)}$$

$x = -3$

$x - 3 = 0$

$x = 3$

Identify holes in the graph of $f(x) = \frac{x^2 + x - 6}{x - 2}$. Then graph.

$$\frac{(x+3)(x-2)}{(x-2)}$$

V.A.

$x = 2$

Homework:

p. 597 #17-31, 39-41, 45,