Warm Up:

Divide. Assume that all expressions are defined.

$$\frac{x^4 - 9x^2}{x^2 - 4x + 3} \div \frac{x^4 + 2x^3 - 8x^2}{x^2 - 16}$$

$$\frac{x^2(x^2-9)}{(x-3)(x-1)} \div \frac{x^2(x^2+2x-8)}{(x-4)(x+4)}$$

$$\frac{(x^2)\cancel{(x-3)}(x+3)}{\cancel{(x-3)}(x-1)} \div \frac{(x^2)\cancel{(x+4)}(x-2)}{(x-4)\cancel{(x+4)}}$$

"Nothing is impossible, the word itself says 'I'm possible'!"

-Audrey Hepburn

18. $\dfrac{4x-8}{x^2-2x}$

$\dfrac{4(x-2)}{x(x-2)}$

$\dfrac{4}{x}$; $x \neq 0$ and $x \neq 2$

19. $\dfrac{8x-4}{2x^2+9x-5}$

$\dfrac{4(2x-1)}{(x+5)(2x-1)}$

$\dfrac{4}{x+5}$; $x \neq -5$ and $x \neq \dfrac{1}{2}$

20. $\dfrac{x^2-36}{x^2-12x+36}$

$\dfrac{(x+6)(x-6)}{(x-6)^2}$

$\dfrac{x+6}{x-6}$; $x \neq 6$

24. $\dfrac{x^2y}{4xy} \cdot \dfrac{x}{6} \cdot \dfrac{3y^5}{x^4}$

$\dfrac{x^2}{24} \cdot \dfrac{3y^5}{x^4}$

$\dfrac{y^5}{8x^2}$

26. $\dfrac{x^2-2x-8}{9x^2-16} \cdot \dfrac{3x^2+10x+8}{x^2-16}$

$\dfrac{(x-4)(x+2)}{(3x+4)(3x-4)} \cdot \dfrac{(3x+4)(x+2)}{(x+4)(x-4)}$

$\dfrac{(x+2)^2}{(3x-4)(x+4)}$

28. $\dfrac{4x^2+15x+9}{8x^2+10x+3} \div \dfrac{x^2+4x}{2x+1}$

$\dfrac{(4x+3)(x+3)}{(4x+3)(2x+1)} \cdot \dfrac{2x+1}{x(x+4)}$

$\dfrac{x+3}{x(x+4)}$

30. $\dfrac{x+2}{x-4} \div \dfrac{1}{3x-12}$

$\dfrac{x+2}{x-4} \cdot 3(x-4)$

$3(x+2)$

$3x+6$

32. $\dfrac{3x^2+10x+8}{-x-2} = -2$

$\dfrac{(3x+4)(x+2)}{-(x+2)} = -2$

$-(3x+4) = -2$

$3x+4 = 2$

$x = -\dfrac{2}{3}$

33. $\dfrac{x^2-9}{x-3} = 5$

$\dfrac{(x+3)(x-3)}{x-3} = 5$

$x+3 = 5$

$x = 2$

34. $\dfrac{x^2 + 3x - 28}{(x + 7)(x - 4)} = -11$

$\dfrac{(x + 7)(x - 4)}{(x + 7)(x - 4)} = -11$

$1 = -11$

no solution

45. Student A; the student didn't leave a 1 in the numerator.

8-3 Adding and Subtracting Rational Expressions

Objectives

Add and subtract rational expressions.

Simplify complex fractions.

8-3 Adding and Subtracting Rational Expressions

Adding and subtracting rational expressions is similar to adding and subtracting fractions. To add or subtract rational expressions with like denominators, add or subtract the numerators and use the same denominator.

$$\frac{1}{5} + \frac{3}{5} = \frac{4}{5} \qquad \frac{6}{7} - \frac{4}{7} = \frac{2}{7}$$

To add or subtract fractions we need a common denominator. This is the same for rational expressions. For now they will give us one and we will discover how to find one later. But for now concentrate on the signs, either add or subtract. Remember if you subtract that minus sign applies to all of an expression not just the first term...

Add or subtract. Identify any x-values for which the expression is undefined.

$$\frac{x-3}{x+4} + \frac{x-2}{x+4}$$

$$= \frac{2x-5}{x+4}$$

Add or subtract. Identify any x-values for which the expression is undefined.

$$\frac{3x-4}{x^2+1} - \frac{6x+1}{x^2+1}$$

$$\frac{-3x-5}{x^2+1}$$

Add or subtract. Identify any x-values for which the expression is undefined.

$$\frac{6x + 5}{x^2 - 3} + \frac{3x - 1}{x^2 - 3}$$

$$\frac{9x+4}{x^2-3}$$

Add or subtract. Identify any x-values for which the expression is undefined.

$$\frac{3x^2 - 5}{3x - 1} - \frac{2x^2 - 3x - 2}{3x - 1}$$

To find the least common multiple of a polynomial first list out all of its factors and then find the similar factors, list those only once and list all of the other factors. Multiply out...

Before we do this with variables and numbers lets just look at how we would think about it with numbers. What is the least common multiple of 21 and 27? First we factor 21 and 27 as small as possible.

Second we combine all the terms necessary to find a multiple of the two.

Find the least common multiple for each pair.

A. $4x^2y^3$ and $6x^4y^5$

$\cancel{2}\cdot\cancel{2}\cdot\cancel{x^2}\cdot\cancel{y^3}$ $\quad$ $\cancel{2}\cdot\cancel{3}\cdot\cancel{x^4}\cancel{y^5}$

$2\cdot 2\cdot 3\cdot x^4\cdot y^5 = 12x^4y^5$

B. $x^2 - 2x - 3$ and $x^2 - x - 6$

$\cancel{(x-3)}\cancel{(x+1)}$ $\quad$ $\cancel{(x-3)}\cancel{(x+2)}$

$(x-3)(x+1)(x+2)$

Find the least common multiple for each pair.

a. $4x^3y^7$ and $3x^5y^4$

$\cancel{2}\cdot\cancel{2}\cancel{x^3}\cancel{y^7}$ $\quad$ $\cancel{3}\cancel{x^5}\cancel{y^4}$

$\cancel{2\cdot 2\cdot 3}\ 12\ x^5y^7$

b. $x^2 - 4$ and $x^2 + 5x + 6$

$\cancel{(x-2)}\cancel{(x+2)}$ $\quad$ $(x+3)\cancel{(x+2)}$

$(x-2)(x+2)(x+3)$

To add rational expressions with unlike denominators, rewrite both expressions with the LCD. This process is similar to adding fractions.

First find the least common multiple. Apply that to top and bottom of fraction. Add them and then cancel things before simplifying!

After it is completely simplified is when you find the undefined x-values for the problem.

Add. Identify any *x*-values for which the expression is undefined.

$$\frac{x-3}{x^2+3x-4} + \frac{2x}{x+4}$$

LCM: $(x+4)(x-1)$

$$\frac{x-3}{(x+4)(x-1)} + \frac{2x}{(x+4)}$$

$$\frac{x-3}{(x+4)(x-1)} + \frac{\overset{2x^2-2x}{\cancel{2x\,(x-1)}}}{(x+4)(x-1)}$$

$$\frac{2x^2-x-3}{(x+4)(x-1)}$$

Add. Identify any *x*-values for which the expression is undefined.

$$\frac{x}{x+2} + \frac{-8}{x^2-4}$$

LCM: $(x+2)(x-2)$

$$\frac{x\,(x-2)}{x+2\,(x-2)} + \frac{-8}{(x+2)(x-2)}$$

$x = -2$ or 2

$$\frac{x^2-2x}{(x+2)(x-2)} + \frac{-8}{(x+2)(x-2)}$$

$$\frac{x^2-2x-8}{(x+2)(x-2)}$$

Add. Identify any x-values for which the expression is undefined.

$$\frac{x}{x+3} + \frac{2x+6}{x^2+6x+9}$$

Subtract $\frac{2x^2-30}{x^2-9} - \frac{x+5}{x+3}$. Identify any x-values for which the expression is undefined.

Subtract $\frac{3x-2}{2x+5} - \frac{2}{5x-2}$. Identify any x-values for which the expression is undefined.

LCM: $(2x+5)(5x-2)$

$$\frac{3x-2\,(5x-2)}{2x+5\,(5x-2)} - \frac{2\,(2x+5)}{5x-2\,(2x+5}$$

$$\frac{15x^2-16x+4}{(2x+5)(5x-2)} - \frac{4x+10}{(5x-2)(2x+5)}$$

$$\frac{15x^2-20x-6}{(5x-2)(2x+5)}$$

Subtract $\frac{2x^2+64}{x^2-64} - \frac{x-4}{x+8}$. Identify any x-values for which the expression is undefined.

Some rational expressions are *complex fractions.* A **complex fraction** contains one or more fractions in its numerator, its denominator, or both. Examples of complex fractions are shown below.

$$\frac{x+2}{\frac{3}{x}} \qquad \frac{1+\frac{1}{x}}{4x+5} \qquad \frac{\frac{x+3}{x}}{\frac{x+4}{7x}}$$

Recall that the bar in a fraction represents division. Therefore, you can rewrite a complex fraction as a division problem and then simplify. You can also simplify complex fractions by using the LCD of the fractions in the numerator and denominator.

Simplify. Assume that all expressions are defined.

$$\frac{\frac{x+2}{x-1}}{\frac{x-3}{x+5}}$$

$$\frac{x+2}{x-1} \div \frac{x-3}{x+5}$$

$$\frac{x+2}{x-1} \cdot \frac{x+5}{x-3}$$

$$\frac{x^2+7x+10}{x^2-4x+3}$$

Simplify. Assume that all expressions are defined.

$$\frac{\frac{3}{x}+\frac{x}{2}}{\frac{x-1}{x}}$$

$$\frac{3}{x}+\frac{x}{2} \div \frac{x-1}{x}$$

$$\left(\frac{3}{x}+\frac{x}{2}\right)\cdot\frac{x}{x-1}$$

$$\frac{3x}{x^2-x}+\frac{x^2}{2x-2}$$

$$\frac{3}{x-1}+\frac{x^2}{2x-2}$$

Simplify. Assume that all expressions are defined.

$$\frac{\frac{x+1}{x^2-1}}{\frac{x}{x-1}}$$

$$\frac{x+1}{(x+1)(x-1)} \div \frac{x}{x-1}$$

$$\frac{\cancel{(x+1)}\,1}{\cancel{(x+1)}\cancel{(x-1)}}\cdot\frac{\cancel{(x-1)}}{x}$$

$$=\frac{1}{x}$$

Simplify. Assume that all expressions are defined.

$$\frac{\frac{20}{x-1}}{\frac{6}{3x-3}}$$

$$\frac{20}{x-1} \div \frac{6}{3x-3}$$

$$\frac{\overset{10}{\cancel{20}}}{\cancel{x-1}} \cdot \frac{\cancel{3}\cancel{(x-1)}}{\underset{1}{\cancel{2}\cancel{6}}}$$

$$=10$$

Simplify. Assume that all expressions are defined.

$$\frac{\frac{1}{x}+\frac{1}{2x}}{\frac{x+4}{x-2}}$$

Homework:

p. 588 # 17-30, 34-40(evens), 47