Warm up:

Write out the logarithmic properties (from the other days) by giving an example of each.

$\log 4 + \log 5 = \log 20$

$\log 100 - \log 25 = \log 4$

$3 \log 3 = \log 3^3 = \log 27$

9. $\log_2 (7x + 1) = \log_2 (2 - x)$

$7x + 1 = 2 - x$

$8x = 1$

$x = \frac{1}{8}$

10. $\log_6 (2x + 3) = 3$

$6^{\log_6 (2x+3)} = 6^3$

$2x + 3 = 216$

$2x = 213$

$x = 106.5$

11. $\log 72 - \log\left(\frac{2x}{3}\right) = 0$

$\log 72 = \log\left(\frac{2x}{3}\right)$

$72 = \frac{2x}{3}$

$216 = 2x$

$108 = x$

12. $\log_3 x^9 = 12$

$9 \log_3 x = 12$

$\log_3 x = \frac{4}{3}$

$x = 3^{\frac{4}{3}}$

$x \approx 4.33$

21.
$$\begin{aligned} 2^{x-1} &= \frac{1}{64} \\ 2^{x-1} &= 2^{-6} \\ x - 1 &= -6 \\ x &= -5 \end{aligned}$$

22.
$$\begin{aligned} \left(\frac{1}{4}\right)^{x} &= 8^{x-1} \\ \left(2^{-2}\right)^{x} &= \left(2^{3}\right)^{x-1} \\ 2^{-2x} &= 2^{3x-3} \\ -2x &= 3x - 3 \\ -5x &= -3 \\ x &= 0.6 \end{aligned}$$

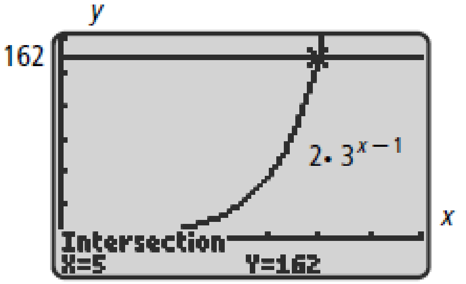
23.
$$\begin{aligned} \left(\frac{1}{5}\right)^{x-2} &= 125^{\frac{x}{2}} \\ \left(5^{-1}\right)^{x-2} &= \left(5^{3}\right)^{\frac{x}{2}} \\ 5^{2-x} &= 5^{\frac{3}{2}x} \\ 2 - x &= \frac{3}{2}x \\ 2 &= \frac{5}{2}x \\ x &= 0.8 \end{aligned}$$

24.
$$\begin{aligned} \left(\frac{1}{2}\right)^{-x} &= 1.6 \\ \log\left(\frac{1}{2}\right)^{-x} &= \log 1.6 \\ -x \log\frac{1}{2} &= \log 1.6 \\ x &= -\frac{\log 1.6}{\log\frac{1}{2}} \\ x &\approx 0.678 \end{aligned}$$

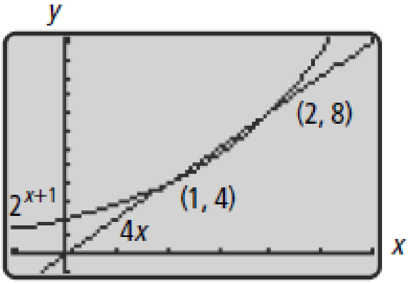
25.
$$\begin{aligned} (1.5)^{x-1} &= 14.5 \\ \log(1.5)^{x-1} &= \log 14.5 \\ (x-1)\log 1.5 &= \log 14.5 \\ x - 1 &= \frac{\log 14.5}{\log 1.5} \\ x &= \frac{\log 14.5}{\log 1.5} + 1 \\ x &\approx 7.595 \end{aligned}$$

26.
$$\begin{aligned} 3^{\frac{x}{2}+1} &= 12.2 \\ \log 3^{\frac{x}{2}+1} &= \log 12.2 \\ \left(\frac{x}{2}+1\right)\log 3 &= \log 12.2 \\ \frac{x}{2} + 1 &= \frac{\log 12.2}{\log 3} \\ x &= 2\left(\frac{\log 12.2}{\log 3} - 1\right) \\ x &\approx 2.554 \end{aligned}$$

34. $x = 5$

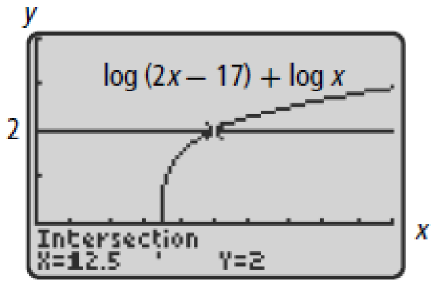


35. $x < 1$



Answer
$x < 1$ or $x > 2$

36. $x \geq 12.5$



38. $5^{2x} = 100$

$$\log 5^{2x} = \log 100$$

$$2x \log 5 = 2$$

$$x \log 5 = 1$$

$$x = \frac{1}{\log 5} \approx 1.43$$

44. The student solved $\log(x + 4) = 8$.

26. $3^{\frac{x}{2}+1} = 12.2$

$\log 3^{\frac{x}{2}+1} = \log 12.2$

$\frac{(\frac{x}{2}+1)\cancel{\log 3}}{\cancel{\log 3}} = \frac{\log 12.2}{\log 3}$

$\frac{x}{2}+\cancel{1} = \frac{\log 12.2}{\log 3}$

$-1 \qquad -1$

$\cancel{2}\cdot\frac{x}{\cancel{2}} = \frac{\log 12.2}{\log 3} - 1 \cdot 2$

$x = 2\left(\frac{\log 12.2}{\log 3} - 1\right)$

$x \approx 2.55$

Throughout all of math there are certain numbers that just "show up". We know about pi, 3.1415..., it helps describe circles and other geometry. There is also the Fibonacci Numbers: 1, 1, 2, 3, 5, 8, ... which actually show up throughout the world in nature.

Today we will be studying e, another number so important it gets a letter. It appears in exponentials (hence the e) and is referred to as the natural base.

7-6 The Natural Base, *e*

Objectives

Use the number *e* to write and graph exponential functions representing real-world situations.

Solve equations and problems involving *e* or natural logarithms.

7-6 The Natural Base, *e*

Vocabulary

natural logarithm

natural logarithmic function

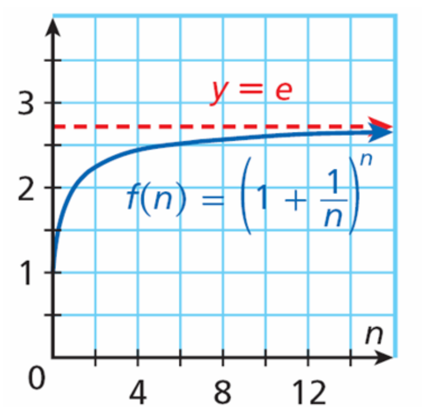
7-6 The Natural Base, *e*

Recall the *compound interest formula* $A = P(1 + \frac{r}{n})^{nt}$, where A is the amount, P is the principal, r is the annual interest, n is the number of times the interest is compounded per year and t is the time in years.

Suppose that \$1 is invested at 100% interest ($r = 1$) compounded n times for one year as represented by the function $f(n) = P(1 + \frac{1}{n})^n$.

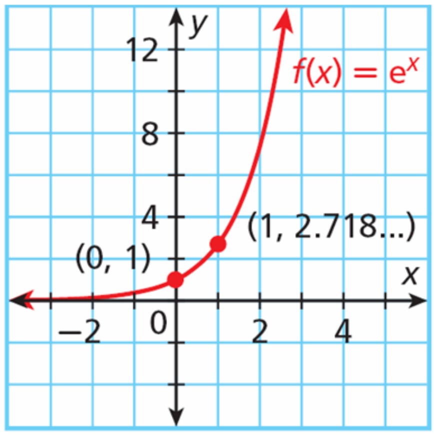
7-6 The Natural Base, *e*

As n gets very large, interest is *continuously compounded*. Examine the graph of $f(n) = (1 + \frac{1}{n})^n$. The function has a horizontal asymptote. As n becomes infinitely large, the value of the function approaches approximately 2.7182818…. This number is called e. Like π, the constant e is an irrational number.



7-6 The Natural Base, *e*

Exponential functions with *e* as a base have the same properties as the functions you have studied. The graph of $f(x) = e^x$ is like other graphs of exponential functions, such as $f(x) = 3^x$.

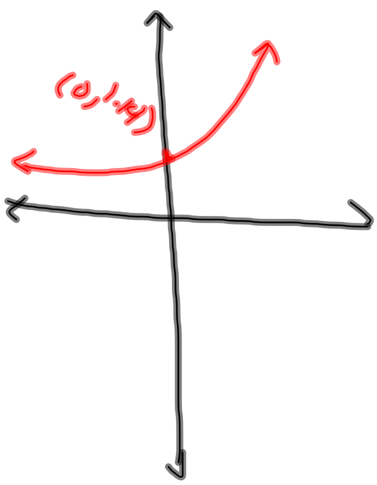


The domain of $f(x) = e^x$ is all real numbers. The range is $\{y | y > 0\}$.

7-6 The Natural Base, *e*

Example 1: Graphing Exponential Functions

Graph $f(x) = e^{x-2} + 1$. $Y_1 = e \wedge (x-2)+1$



$e^{0-2} + 1$

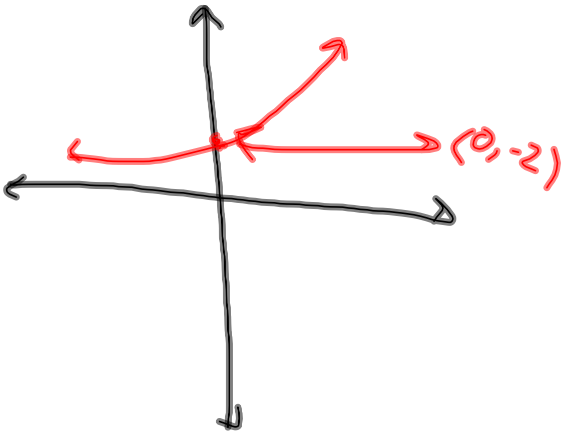
$e^{-2} + 1$

7-6 The Natural Base, *e*

Check It Out! Example 1

Graph $f(x) = e^x - 3$.

$e^0 - 3$



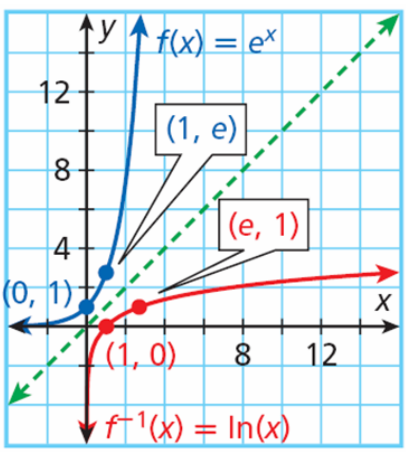
$\log_{10}$ $\log_e$ $\ln = \ln_e$

7-6 The Natural Base, *e*

A logarithm with a base of *e* is called a **natural logarithm** and is abbreviated as "ln" (rather than as $\log_e$). Natural logarithms have the same properties as log base 10 and logarithms with other bases.

$\ln 3 + \ln 4 = \ln 12$

The **natural logarithmic function** $f(x) = \ln x$ is the inverse of the natural exponential function $f(x) = e^x$.



Holt Algebra 2 Copyright © by Holt, Rinehart and Winston. All Rights Reserved.

Whenever we see ln we are dealing with a logarithm that has a base of e. Nothing changes except the letters used to write it. The hook trick still works and the properties still apply.

You aren't really learning anything new just how to apply what you know to other things.

$\ln 20 - \ln 10 = \ln 2$

Remember, to solve follow the steps:

1) Simplify with log/ln properties.
2) Ln and e will cancel...
3) Simplify again

7-6 The Natural Base, *e*

Example 2: Simplifying Expression with *e* or ln

Simplify.

A. $\cancel{\ln e}^{0.15t}$

$.15t$

B. $e^{3\ln(x+1)}$

$\cancel{e}^{\cancel{\ln}(x+1)^3}$

$(x+1)^3$

C. $\ln e^{2x} + \ln e^{x}$

$\ln e^{3x}$

$3x\,\cancel{\ln e}$

$3x$

7-6 The Natural Base, *e*

Check It Out! Example 2

Simplify.

a. $\ln e^{3.2}$

$3.2\,\cancel{\ln e}$

3.2

b. $e^{2\ln x}$

$\cancel{e}^{\cancel{\ln} x^2}$

x^2

c. $\ln e^{x+4y}$

$x+4y\,\cancel{\ln e}$

$x+4y$

You have seen in the past interest formulas that are compounded at a certain rate, such as making money on a bank account once a month. The cool thing with e is that is allows us to start compounding things continuously, or all the time. This comes in helpful when we look at things such as bacteria and cultures.

Without e it would be impossible to diagnose many diseases including flu, hepatitis, strep, mono, pneumonia, etc.

7-6 The Natural Base, *e*

The formula for continuously compounded interest is $A = Pe^{rt}$, where A is the total amount, P is the principal, r is the annual interest rate, and t is the time in years.

7-6 The Natural Base, *e*

Example 3: Economics Application

What is the total amount for an investment of $500 invested at 5.25% for 40 years and compounded continuously?

$A = Pe^{rt}$

$A = 500e^{.0525 \cdot 40}$

$A = 4083.08$

7-6 The Natural Base, *e*

Check It Out! Example 3

What is the total amount for an investment of $100 invested at 3.5% for 8 years and compounded continuously?

$A = Pe^{rt}$

$100e^{.035 \cdot 8}$

$= 132.31$

7-6 The Natural Base, *e*

The *half-life* of a substance is the time it takes for half of the substance to breakdown or convert to another substance during the process of decay. Natural decay is modeled by the function below.

N_0 is the initial amount (at $t = 0$). k is the decay constant.

$$N(t) = N_0 e^{-kt}$$

$N(t)$ is the amount remaining. t is the time.

Holt Algebra 2

When given half life problems there are a few steps to take. Follow these and get them all right!

1) Find the decay constant, k. This is done by using the half life.
Therefore equation will be $.5=1e^{-kt}$ where t is the half life given. Remember e is just a number.
2) Use the constant and the information given to find answer.

7-6 The Natural Base, *e*

Example 4: Science Application

Pluonium-239 (Pu-239) has a half-life of 24,110 years. How long does it take for a 1 g sample of Pu-239 to decay to 0.1 g?

$.5 = 1e^{-k \cdot 24110}$

$\ln .5 = \ln e^{-k \cdot 24110}$

$\frac{\ln .5}{-24110} = \frac{-k \cdot 24110 \cancel{\ln e}}{-24110}$

$k = \frac{-\ln .5}{24110}$

$k = .0000287$

$k = .0000287$

$1g \rightarrow .1g$

$.1 = 1e^{-.0000287t}$

$\ln .1 = \ln e^{-.0000287t}$

$\frac{\ln .1}{-.0000287t} = \frac{-.0000287t \cancel{\ln e}}{}$

$t = \frac{\ln .1}{-.0000287}$

$t = 80229.45 \text{ yrs}$

7-6 The Natural Base, *e*

Check It Out! Example 4

Determine how long it will take for 650 mg of a sample of chromium-51 which has a half-life of about 28 days to decay to 200 mg.

$$.5 = 1e^{-k\cdot 28}$$

$$\ln .5 = \ln e^{-k\cdot 28}$$

$$\frac{\ln .5 = -k\cdot 28 \cancel{\ln e}}{-28}$$

$$k = \frac{\ln .5}{-28}$$

$$k = .0247$$

$$\frac{200}{650} = \frac{\cancel{650}e^{-.0247t}}{\cancel{650}}$$

$$.308 = e^{-.0247t}$$

$$\ln .308 = \ln e^{-.0247t}$$

$$\frac{\ln .308 = -.0247t \cancel{\ln e}}{-.0247}$$

$$t = \frac{\ln .308}{-.0247}$$

$$t = 47.68 \text{ days}$$

Check yourself:

A new element has been found that has a half life of 6 years. Use this information to find out how long it will take for 7 grams of the element to decay to 2 grams.

$.5 = 1e^{-k \cdot 6}$

$k = \frac{\ln .5}{-6}$

$k = .116$

$2 = 7e^{-.116t}$

$\frac{\ln \frac{2}{7}}{-.116} = t$

$10.8 = t$

Homework
p. 534 #14, 15, 17-22 (22 has 3 parts), 31-34

Presentation: 22