$x^2 = 16$

Warm Up:
Solve the following:

1. $\log_{16} x = \frac{3}{2}$ $16^{\frac{3}{2}} = x$

2. $\log_x 1.331 = 3$ $\sqrt[3]{x^3} = \sqrt[3]{1.331}$

3. $\log 10{,}000 = x$

$10^x = 10000$

Quote for this week:

"Even a mistake may turn out to be the one thing necessary to a worthwhile achievement."

-Henry Ford

7-5 Exponential and Logarithmic Equations and Inequalities

Objectives

Solve exponential and logarithmic equations and equalities.

Solve problems involving exponential and logarithmic equations.

$4^x = 40$

7-5 Exponential and Logarithmic Equations and Inequalities

Vocabulary

exponential equation

logarithmic equation

Holt Algebra 2
Copyright © by Holt, Rinehart and Winston. All Rights Reserved.

$y = ab^x$

An **exponential equation** is an equation containing one or more expressions that have a variable as an exponent. To solve exponential equations:

Take the logarithm of both sides...

$\log_3 4^5$

We have found a logarithmic property that says:

$\log_a b^x = x \log_a b$

So if we had an equation we could use that same property to help isolate the x.
REMEMBER: if you take the log of one side you have to take the log of the other side...

Ex: $4^x = 56$ solve for x.

$\log 4^x = \log 56$

$\frac{x \log 4}{\log 4} = \frac{\log 56}{\log 4}$

$x = 2.9$

Steps for getting x out of the exponent:
1) take the log of both sides
2) put the exponent (x) out front
3) get x by itself

Remember the log is just a number, don't let the letters confuse you. You can add, subtract, multiply and divide with logs.
Ex: log 10 = 1

If you don't like taking the log of both sides there are some problems that will allow you to get to the same base.

If you can make two exponents base the same then the powers will be equal.

Ex: $3^x = 27^{x-2}$

27 is just 3^3 so we could switch 27 with 3^3 to get $3^x = (3^3)^{x-2}$.

We then use our exponent properties to get $3^x = 3^{3x-6}$.

Now since our bases are equal we can set the powers equal to each other and solve.

$x = 3x - 6$

$-2x = -6$

$x = 3$

Steps for same base solving:

1) get the same base by raising a base to a power Ex: $27 = 3^3$
2) Use exponent properties to simplify the new exponent.
3) Set the powers equal to each other and solve for x.

7-5 Exponential and Logarithmic Equations and Inequalities

Example 1A: Solving Exponential Equations

Solve and check.

$9^{8-x} = 27^{x-3}$

$(3^2)^{(8-x)} = (3^3)^{x-3}$

$3^{16-2x} = 3^{3x-9}$

$16 - 2x = 3x - 9$

$+2x \quad +2x$

$16 - 5x - 9$

$+9 \quad +9$

$\frac{25}{5} = \frac{5x}{5}$

$x = 5$

7-5 Exponential and Logarithmic Equations and Inequalities

Example 1B: Solving Exponential Equations

Solve and check.

$4^{x-1} = 5$

$\log 4^{x-1} = \log 5$

$(x-1) \cdot \log 4 = \log 5$

$\frac{(x-1) \cdot \log 4}{\log 4} = \frac{\log 5}{\log 4}$

$x - 1 = \frac{\log 5}{\log 4}$

$+1 \quad +1$

$x = \frac{\log 5}{\log 4} + 1$

$x = 2.16$

7-5 Exponential and Logarithmic Equations and Inequalities

Check It Out! Example 1a

Solve and check.

$3^{2x} = 27$

$3^{2x} = 3^3$

$2x = 3$

$x = \frac{3}{2}$

7-5 Exponential and Logarithmic Equations and Inequalities

Check It Out! Example 1b

Solve and check.

$7^{-x} = 21$

$\log 7^{-x} = \log 21$

$\frac{-x \cdot \log 7}{\log 7} = \frac{\log 21}{\log 7}$

$x = \frac{\log 21}{\log 7} \cdot -1$

$x = -\frac{\log 21}{\log 7} = -1.56$

Holt Algebra 2

7-5 Exponential and Logarithmic Equations and Inequalities

Check It Out! Example 1c

Solve and check.

$2^{3x} = 15$

Challenge your partner:
Write them an equation with x in the exponent and have them solve.
Be sure to check each other's answers

7-5 Exponential and Logarithmic Equations and Inequalities

Example 2: Biology Application

Suppose a bacteria culture doubles in size every hour. How many hours will it take for the number of bacteria to exceed 1,000,000?

$2^x = 1000000$

$\log 2^x = \log 1000000$

$x = \frac{\log 1000000}{\log 2}$

$x = 19.93$

7-5 Exponential and Logarithmic Equations and Inequalities

$\log_{10} x = 14$

A **logarithmic equation** is an equation with a logarithmic expression that contains a variable. You can solve logarithmic equations by using the properties of logarithms.

If $\log_b x = \log_b y$ then $x = y$

You may also have to solve when x is "in" the logarithm. Don't let this scare you, just use that hook rule to get the logarithm out of there and continue to solve like normal.

Ex: $\log_7 (2x-1) = 4$ solve for x

$7^4 = 2x-1$

$+1 \quad +1$

$\frac{7^4+1}{2} = \frac{2x}{2}$

$x = \frac{7^4+1}{2}$

$= 2401.5$

Steps for when x is in the logarithm:
1) Simplify each side as much as possible using logarithm rules.
2) Use the hook rule to write the logarithm as an exponent and get rid of it
3) Solve for x

7-5 Exponential and Logarithmic Equations and Inequalities

Example 3A: Solving Logarithmic Equations

Solve.

$\log_6(2x - 1) = -1$

$6^{-1} = 2x - 1$

$+1 \quad +1$

$\frac{6^{-1}+1}{2} = \frac{2x}{2}$

$x = \frac{(6^{-1}+1)}{2} = .583$

7-5 Exponential and Logarithmic Equations and Inequalities

Example 3B: Solving Logarithmic Equations

Solve.

$\log_4 100 - \log_4(x + 1) = 1$

$\log_4 \frac{100}{x+1} = 1$

$4^1 = \frac{100}{x+1}$

$\frac{4}{1} = \frac{100}{x+1}$

$4(x+1) = 100$

$4x + 4 = 100$

$-4 \quad -4$

$\frac{4x}{4} = \frac{96}{4}$

$x = 24$

7-5 Exponential and Logarithmic Equations and Inequalities

Example 3C: Solving Logarithmic Equations

Solve.

$\log_5 x^4 = 8$

$5^8 = x^4$

$\sqrt[4]{5^8} = \sqrt[4]{x^4}$

$4\sqrt{5^8}$

25

7-5 Exponential and Logarithmic Equations and Inequalities

Example 3D: Solving Logarithmic Equations

Solve.

$\log_{12} x + \log_{12}(x + 1) = 1$

$\log_{12} x(x+1) = 1$

$\log_{12} x^2 + x = 1$

$12^1 = x^2 + x$

$12 = x^2 + x$

$-12 \quad -12$

$0 = x^2 + x - 12$

$0 = (x-3)(x+4)$

$x = 3$ or -4

You may have to solve graphs that involve inequalities. Don't make it difficult, use a calculator!

To do this: put both equations in for y1 and y2 and then look for the solution or where the inequality is true.

Example 4A: Using Tables and Graphs to Solve Exponential and Logarithmic Equations and Inequalities

Use a table and graph to solve $2^{x+1} > 8192x$.

$Y_1 = 2^{x+1}$ 2^(x+1)

$Y_2 = 8192X$

7-5 Exponential and Logarithmic Equations and Inequalities

Example 4B

$\log(x + 70) = 2\log\left(\frac{x}{3}\right)$

$Y_1 = \log(x+70)$

$Y_2 = 2\log\left(\frac{x}{3}\right)$

7-5 Exponential and Logarithmic Equations and Inequalities

Check It Out! Example 4a

Use a table and graph to solve $2^x = 4^{x-1}$.

7-5 Exponential and Logarithmic Equations and Inequalities

Check It Out! Example 4c

Use a table and graph to solve $\log x^2 = 6$.

Homework:
p. 526 #9-12, 21-26, 34-36, 38, 44

Present: 38, 44

7-5 Exponential and Logarithmic Equations and Inequalities

Check It Out! Example 4b

Use a table and graph to solve $2^x > 4^{x-1}$.