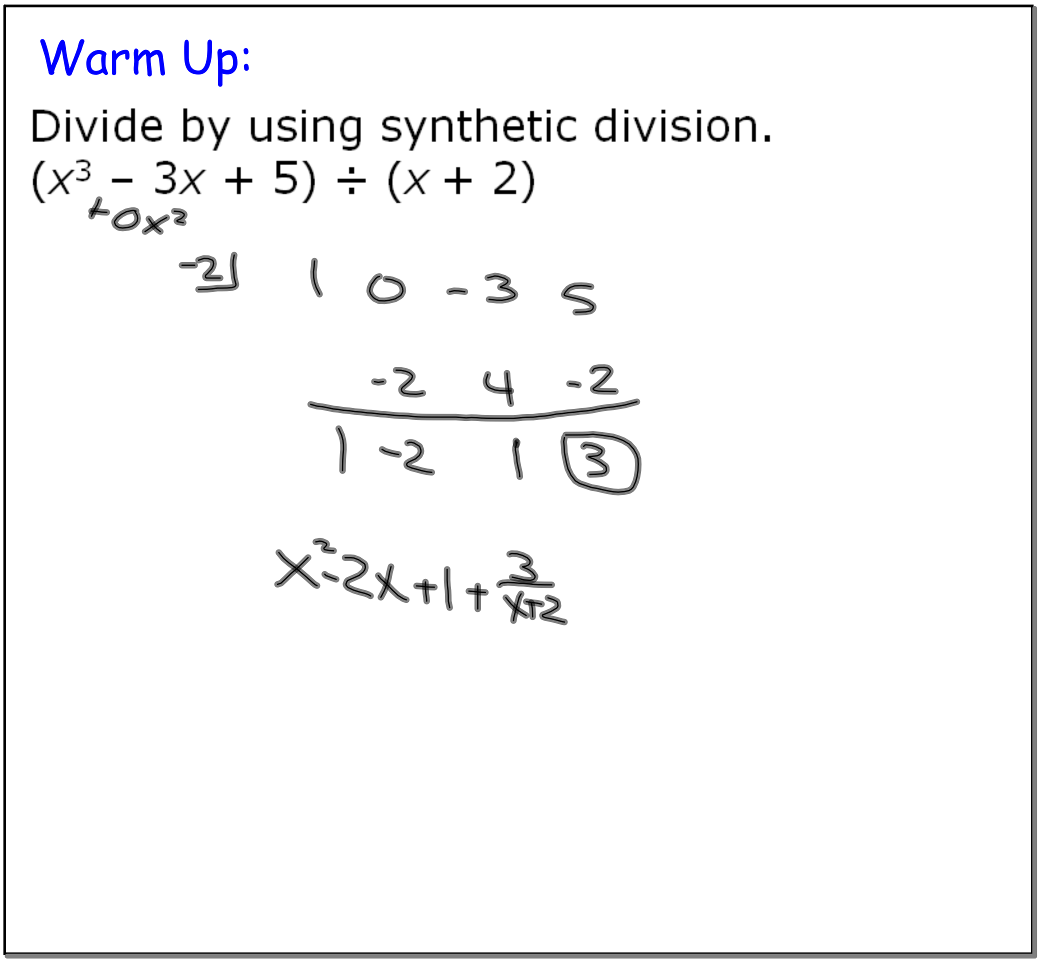


Warm Up:

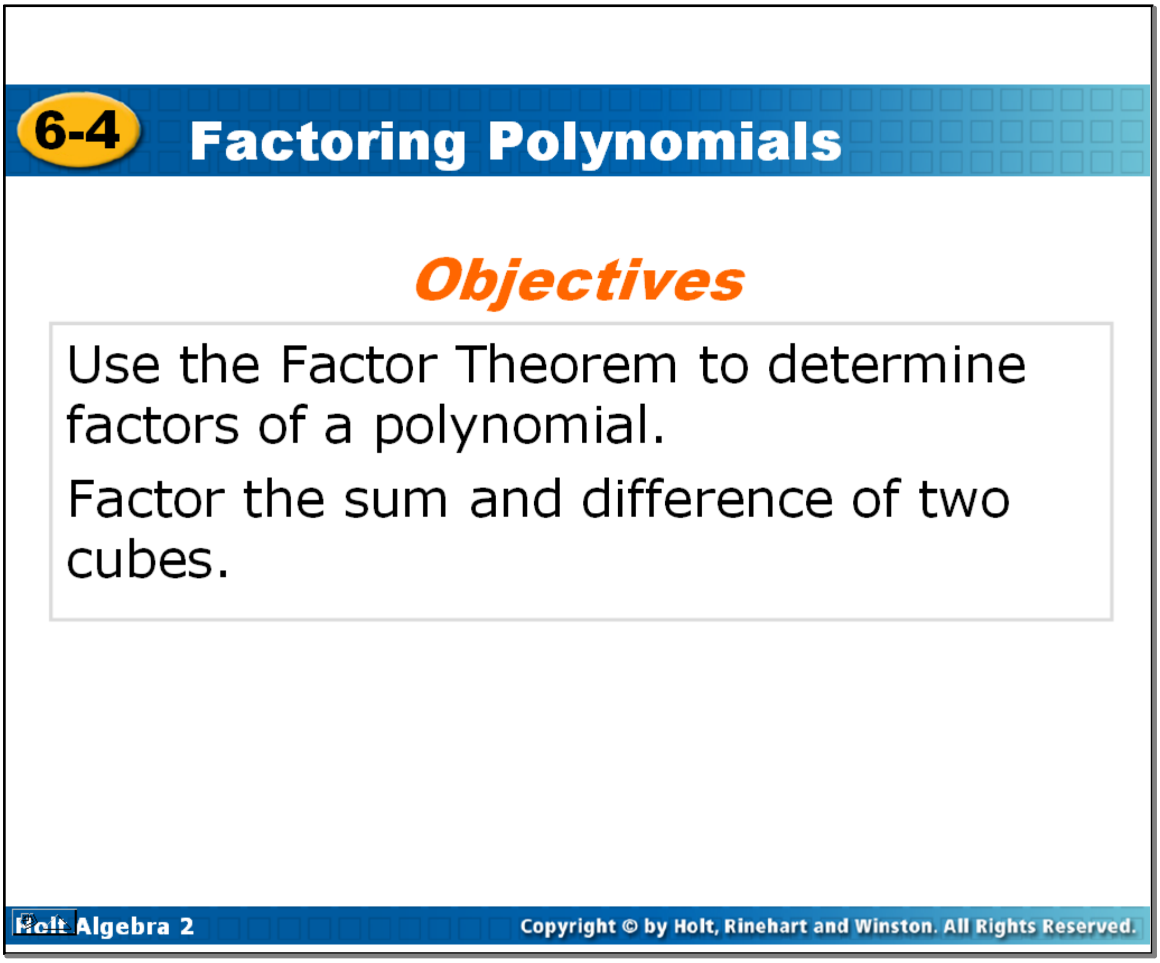
Divide by using synthetic division.

$(x^3 - 3x + 5) \div (x + 2)$

$+0x^2$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -3 & 5 \\ & & -2 & 4 & -2 \\ \hline & 1 & -2 & 1 & \boxed{3} \end{array}$$

$x^2 - 2x + 1 + \frac{3}{x+2}$

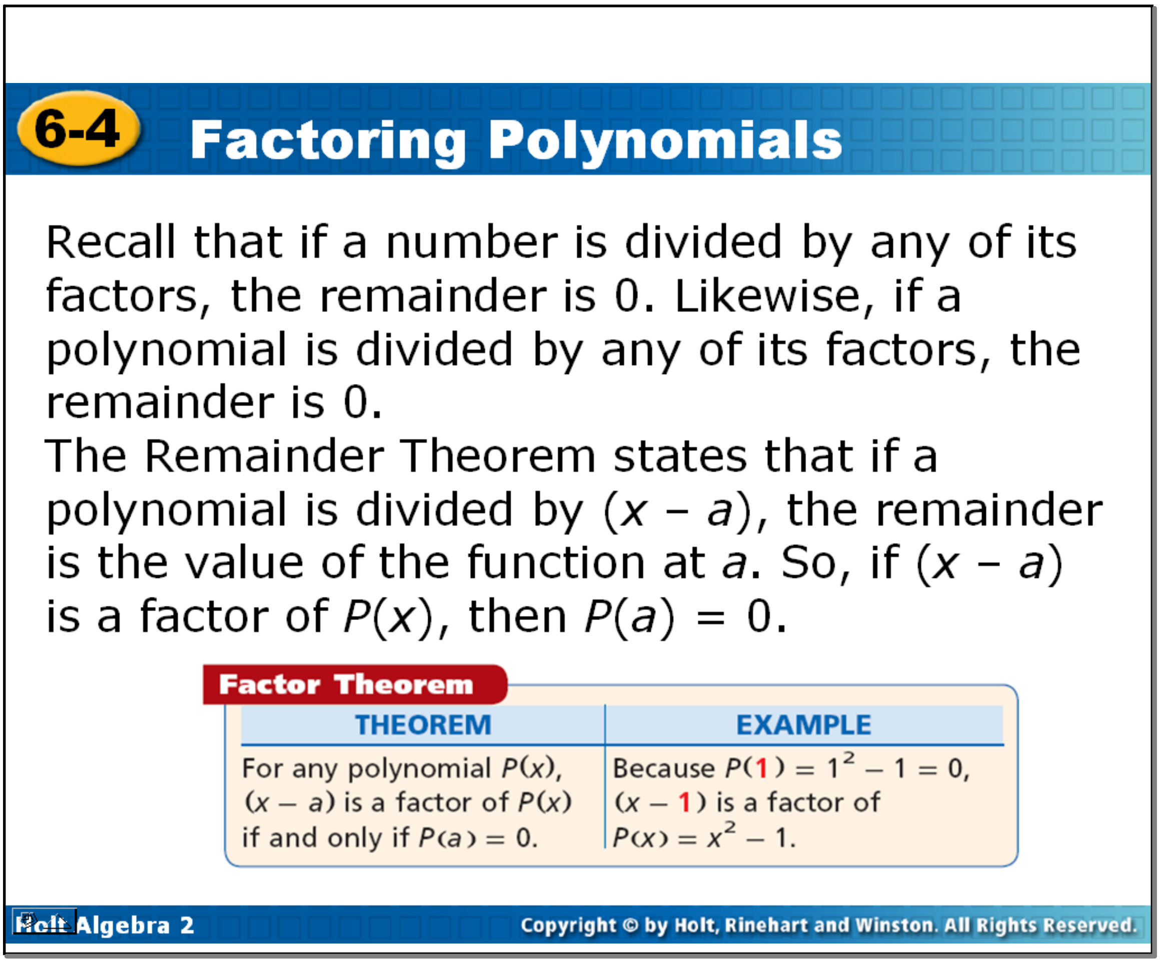


6-4 Factoring Polynomials

Objectives

Use the Factor Theorem to determine factors of a polynomial.

Factor the sum and difference of two cubes.



6-4 Factoring Polynomials

Recall that if a number is divided by any of its factors, the remainder is 0. Likewise, if a polynomial is divided by any of its factors, the remainder is 0.

The Remainder Theorem states that if a polynomial is divided by $(x - a)$, the remainder is the value of the function at a. So, if $(x - a)$ is a factor of $P(x)$, then $P(a) = 0$.

Factor Theorem

THEOREM	EXAMPLE
For any polynomial $P(x)$, $(x - a)$ is a factor of $P(x)$ if and only if $P(a) = 0$.	Because $P(1) = 1^2 - 1 = 0$, $(x - 1)$ is a factor of $P(x) = x^2 - 1$.

Holt Algebra 2

We have to use synthetic substitution again. If we get an answer of 0 then we have a factor for our polynomial.

Determine whether the given binomial is a factor of the polynomial $P(x)$.

A. $(x + 1)$; $(x^2 - 3x + 1)$

$-1\,|\;\; 1 \;\; -3 \;\; 1$

$\quad -1 \;\; 4$

$1 \;\; -4 \;\; \boxed{5}$

Not a factor

B. $(x + 2)$;
$(3x^4 + 6x^3 \overset{+0x^2}{-} 5x - 10)$

$-2\,|\;\; 3 \;\; 6 \;\; 0 \;\; -5 \;\; -10$

$\quad -6 \;\; 0 \;\; 0 \;\; 10$

$3 \;\; 0 \;\; 0 \;\; -5 \;\; \boxed{0}$

Yes, it's a factor

Determine whether the given binomial is a factor of the polynomial $P(x)$.

a. $(x + 2)$; $(4x^2 - 2x + 5)$

$-2\,|\;\; 4 \;\; -2 \;\; 5$

$\quad -8 \;\; 20$

$4 \;\; -10 \;\; 25$

Not a factor

b. $(3x - 6)$;
$(3x^4 - 6x^3 + 6x^2 + 3x - 30)$

$x-2;\; x^4 - 2x^3 + 2x^2 + x - 10$

$2\,|\;\; 1 \;\; -2 \;\; 2 \;\; 1 \;\; -10$

$\quad 2 \;\; 0 \;\; 4 \;\; 10$

$1 \;\; 0 \;\; 2 \;\; 5 \;\; 0$

Yes; factor

To factor polynomials:

1) Group terms that you can pull "stuff" out of.

2) Factor your groups and then "stuff" you pull out.

Factor: $x^3 - x^2 - 25x + 25$.

$(x^3 - x^2) + (-25x + 25)$

$x^2\overset{y}{(x-1)} + -25\overset{y}{(x-1)}$

$(x-1)(x^2-25)$

$(x-1)(x+5)(x-5)$

$x^2y + -25y$

$y(x^2-25)$

Factor: $x^3 - 2x^2 - 9x + 18$.

$$(x^3 - 2x^2) + (-9x + 18)$$

$$x^2(x-2) + -9(x-2)$$

$$(x-2)(x^2-9)$$

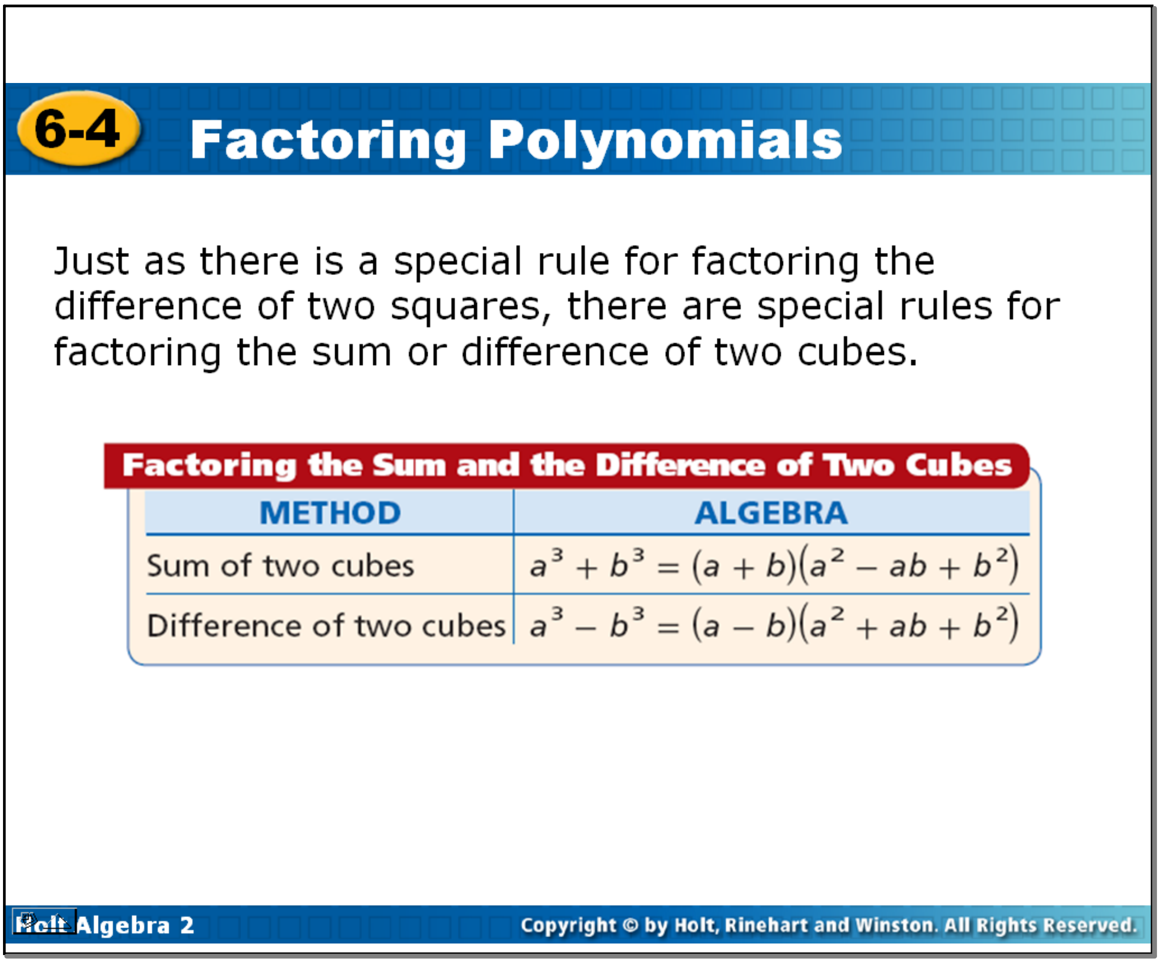
$$(x-2)(x+3)(x-3)$$

Factor: $2x^3 + x^2 + 8x + 4$.

$$(2x^3 + x^2) + (8x + 4)$$

$$x^2(2x+1) + 4(2x+1)$$

$$(2x+1)(x^2+4)$$



6-4 Factoring Polynomials

Just as there is a special rule for factoring the difference of two squares, there are special rules for factoring the sum or difference of two cubes.

Factoring the Sum and the Difference of Two Cubes

METHOD	ALGEBRA
Sum of two cubes	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
Difference of two cubes	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Factor the expression.

$4x^4 + 108x$

$4x(x^3+27)$

a: x
b: 3

$(4x)(x+3)(x^2-3x+9)$

Factor the expression.

$125d^3 - 8$

$a: 5d$

$b: 2$

$(5d-2)(25d^2+10+4)$

Factor the expression.

$8 + z^6$ $(z^2)^3 = z^6$

$a = 2$

$b = z^2$

$(2+z^2)(4-2z^2+z^4)$

Factor the expression.

$2x^5 - 16x^2$

$2x(x^3 - 8)$

$a = x$

$b = 2$

$(2x)(x-2)(x^2+2x+4)$

Homework:

p. 433 #1-15, 17-25, 50-52