Warm Up:

Find the zeros of each function by using the Quadratic Formula.

a b c

1. $f(x) = 3x^2 - 6x - 5$

a b c

2. $g(x) = 2x^2 - 6x + 5$

5-8 Curve Fitting with Quadratic Models

Objectives

Use quadratic functions to model data.

Use quadratic models to analyze and predict.

5-8 Curve Fitting with Quadratic Models

Vocabulary

quadratic model

quadratic regression

5-8 Curve Fitting with Quadratic Models

Recall that you can use differences to analyze patterns in data. For a set of ordered parts with equally spaced x-values, a quadratic function has constant nonzero second differences, as shown below.

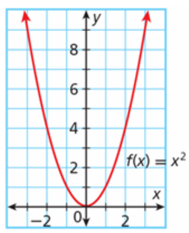
Equally spaced x-values

x	-3	-2	-1	0	1	2	3
$f(x) = x^2$	9	4	1	0	1	4	9

1st differences $\quad -5 \quad -3 \quad -1 \quad 1 \quad 3 \quad 5$

2nd differences $\quad 2 \quad 2 \quad 2 \quad 2 \quad 2$

Constant 2nd differences



Quadratic equations have constant second differences, the reason: x^2. If your data has constant second differences then you are working with a quadratic set of data.

Determine whether the data set could represent a quadratic function. Explain.

x	1	3	5	7	9
y	–1	1	7	17	31

2 6 10 14

4 4 4

Quadratic

Determine whether the data set could represent a quadratic function. Explain.

x	3	4	5	6	7
y	1	3	9	27	81

2 6 18 54

4 12 36

Determine whether the data set could represent a quadratic function. Explain.

x	3	4	5	6	7
y	11	21	35	53	75

10 14 18 22

4 4 4

Determine whether the data set could represent a quadratic function. Explain.

x	10	9	8	7	6
y	6	8	10	12	14

5-8 **Curve Fitting with Quadratic Models**

A **quadratic model** is a quadratic function that represents a real data set. Models are useful for making estimates.

In Chapter 2, you used a graphing calculator to perform a *linear regression* and make predictions. You can apply a similar statistical method to make a quadratic model for a given data set using **quadratic regression**.

5-8 Curve Fitting with Quadratic Models

Helpful Hint

The coefficient of determination R^2 shows how well a quadratic function model fits the data. The closer R^2 is to 1, the better the fit. In a model with $R^2 \approx 0.996$, which is very close to 1, the quadratic model is a good fit.

Like linear regressions we put our independent variable under L1 and our dependent variable under L2. Then use stat-calc-QuadReg to find our equation.

The table shows the cost of circular plastic wading pools based on the pool's diameter. Find a quadratic model for the cost of the pool, given its diameter. Use the model to estimate the cost of the pool with a diameter of 8 ft.

Diameter (ft) x	4	5	6	7
Cost y	$19.95	$20.25	$25.00	$34.95

$y = 2.4x^2 + {-21.6}x + 67.6$

$(8)^2$ (8)

48.4

The tables shows approximate run times for 16 mm films, given the diameter of the film on the reel. Find a quadratic model for the reel length given the diameter of the film. Use the model to estimate the reel length for an 8-inch-diameter film.

Film Run Times (16 mm)		
Diameter (in) x	Reel Length y (ft)	Run Time (min)
5	200	5.55
7	400	11.12
9.25	600	16.67
10.5	800	22.22
12.25	1200	33.33
13.75	1600	44.25

$y = 14.3x^2 + {-112.4}x + 430.1$

$y = 446$

The table shows the prices of an ice cream cake, depending on its side. Find a quadratic model for the cost of an ice cream cake, given the diameter. Then use the model to predict the cost of an ice cream cake with a diameter of 18 in.

Diameter (in.)	Cost
6	$7.50
10	$12.50
15	$18.50

5-9 Operations with Complex Numbers

Objective

Perform operations with complex numbers.

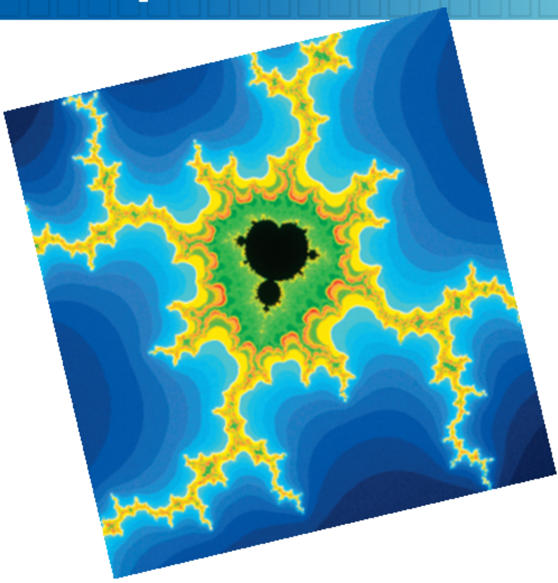
5-9 Operations with Complex Numbers

Vocabulary

complex plane
absolute value of a complex number

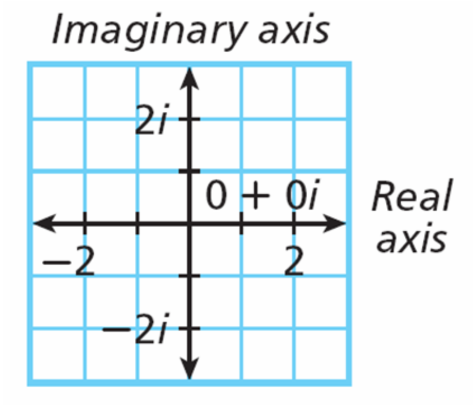
5-9 Operations with Complex Numbers

Just as you can represent real numbers graphically as points on a number line, you can represent complex numbers in a special coordinate plane.



The **complex plane** is a set of coordinate axes in which the horizontal axis represents real numbers and the vertical axis represents imaginary numbers.

5-9 Operations with Complex Numbers



Helpful Hint

The real axis corresponds to the *x*-axis, and the imaginary axis corresponds to the *y*-axis. Think of $a + bi$ as $x + yi$.

Holt Algebra 2

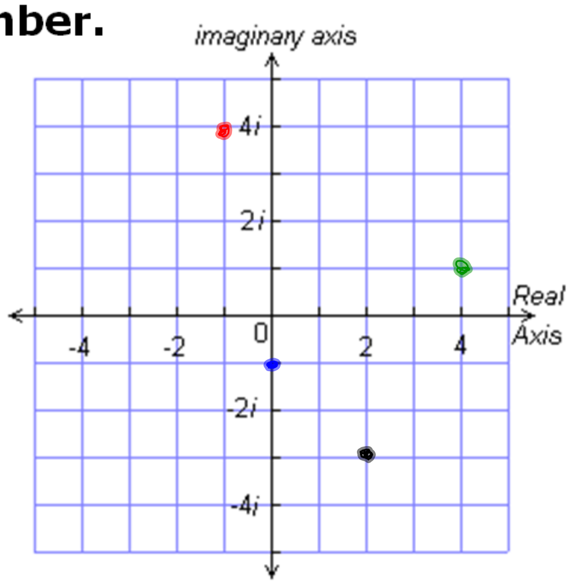
Graph each complex number.

A. $2 - 3i$

B. $-1 + 4i$

C. $4 + i$

D. $-i$



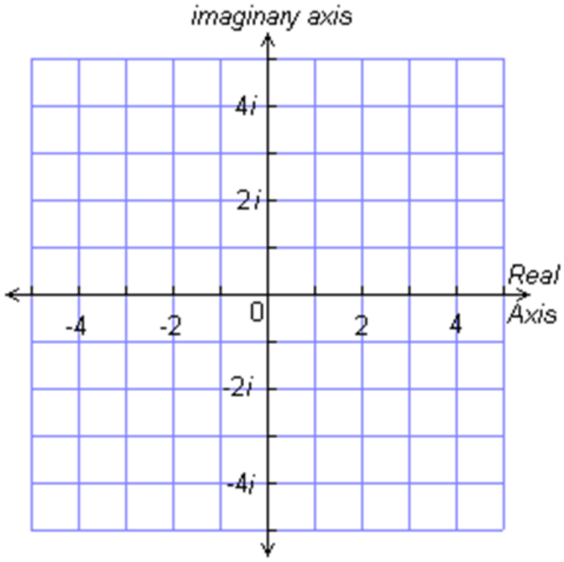
Graph each complex number.

a. $3 + 0i$

b. $2i$

c. $-2 - i$

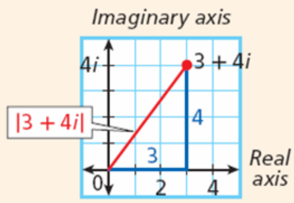
d. $3 + 2i$



5-9 Operations with Complex Numbers

Recall that absolute value of a real number is its distance from 0 on the real axis, which is also a number line. Similarly, the absolute value of an imaginary number is its distance from 0 along the imaginary axis.

Absolute Value of a Complex Number

WORDS	ALGEBRA	EXAMPLE
The **absolute value of a complex number** $a + bi$ is the distance from the origin to the point (a, b) in the complex plane, and is denoted $\lvert a + bi \rvert$.	$\lvert a + bi \rvert = \sqrt{a^2 + b^2}$	 $\lvert 3 + 4i \rvert = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = 5$

To find the absolute value we just square each part, add them together and then find the square root. Just like the distance equation. Since we are using the absolute value there should be no negative number left!

Find each absolute value.

A. $|3 + 5i|$

$\sqrt{3^2+5^2}$

$\sqrt{9+25}$

$\sqrt{34}$

B. $|-13|$

13

C. $|-7i|$

7

Find each absolute value.

a. $|1 - 2i|$ **b.** $\left|-\frac{1}{2}\right|$ **c.** $|23i|$

$\sqrt{1^2+(-2)^2}$

$\sqrt{1+4}$

$\sqrt{5}$

$\frac{1}{2}$

23

5-9 Operations with Complex Numbers

Adding and subtracting complex numbers is similar to adding and subtracting variable expressions with like terms. Simply combine the real parts, and combine the imaginary parts.

The set of complex numbers has all the properties of the set of real numbers. So you can use the Commutative, Associative, and Distributive Properties to simplify complex number expressions.

5-9 Operations with Complex Numbers

Helpful Hint

Complex numbers also have additive inverses. The additive inverse of $a + bi$ is $-(a + bi)$, or $-a - bi$.

When adding or subtracting pair the real parts together and the imaginary parts together to simplify.

Add or subtract. Write the result in the form *a* + *bi*.

$(4 + 2i) + (-6 - 7i)$

$-2 - 5i$

Add or subtract. Write the result in the form *a* + *bi*.

$(5 - 2i) - (-2 - 3i)$

$5 - 2i + 2 + 3i$

$7 + i$

Add or subtract. Write the result in the form *a* + *bi*.

$(1 - 3i) + (-1 + 3i)$

0

Add or subtract. Write the result in the form *a* + *bi*.

$(-3 + 5i) + (-6i)$

Add or subtract. Write the result in the form *a* + *bi*.

$2i - (3 + 5i)$

Add or subtract. Write the result in the form *a* + *bi*.

$(4 + 3i) + (4 - 3i)$

5-9 Operations with Complex Numbers

You can multiply complex numbers by using the Distributive Property and treating the imaginary parts as like terms. Simplify by using the fact $i^2 = -1$.

When multiplying you have to use the distributive property and FOIL. Do not just do first times first and last times last.

$\sqrt{2} \cdot \sqrt{2} = 2$

$\sqrt{3} \cdot \sqrt{3} = 3$

$\sqrt{-1} \cdot \sqrt{-1} = -1$

$i \cdot i = -1$

$i^2 = -1$

Multiply. Write the result in the form $a + bi$.

$-2i(2 - 4i)$

Multiply. Write the result in the form $a + bi$.

$(3 + 6i)(4 - i)$

$12 - 3i + 24i - 6i^2$

$12 + 21i - 6(-1)$

$12 + 21i + 6$

$18 + 21i$

Multiply. Write the result in the form *a* + *bi*.

$(2 + 9i)(2 - 9i)$

$4 - 18i + 18i - 81i^2$

$4 - 81(-1)$

85

Multiply. Write the result in the form *a* + *bi*.

$(-5i)(6i)$

Multiply. Write the result in the form *a* + *bi*.

$2i(3 - 5i)$

Multiply. Write the result in the form *a* + *bi*.

$(4 - 4i)(6 - i)$

$24 - 4i - 24i + 4i^2$

$24 - 28i + 4(-1)$

$20 - 28i$

Multiply. Write the result in the form *a* + *bi*.

$(3 + 2i)(3 - 2i)$

5-9 Operations with Complex Numbers

The imaginary unit *i* can be raised to higher powers as shown below.

Powers of *i*		
$i^1 = i$	$i^5 = i^4 \cdot i = 1 \cdot i = i$	$i^9 = i$
$i^2 = -1$	$i^6 = i^4 \cdot i^2 = 1 \cdot (-1) = -1$	$i^{10} = -1$
$i^3 = i^2 \cdot i = -1 \cdot i = -i$	$i^7 = i^4 \cdot i^3 = 1 \cdot (-i) = -i$	$i^{11} = -i$
$i^4 = i^2 \cdot i^2 = -1 \cdot (-1) = 1$	$i^8 = i^4 \cdot i^4 = 1 \cdot 1 = 1$	$i^{12} = 1$

Helpful Hint

Notice the repeating pattern in each row of the table. The pattern allows you to express any power of *i* as one of four possible values: *i*, −1, −*i*, or 1.

If you take the power that i is being raised to and divide it by 4, if you get a remainder of:

0: It simplifies to 1

1: It simplifies to i

2: It simplifies to -1

3: It simplifies to -i

Simplify –6*i*^{14}.

$-6 \cdot -1$

6

$$\begin{array}{r} 3 \\ 4\overline{)14} \\ 12 \\ \hline R{:}\ 2 \end{array}$$

Simplify i^{63}.

$-i$

$$\begin{array}{r} 15 \\ 4\overline{)63} \\ 4\downarrow \\ \hline 23 \\ 20 \\ \hline R:3 \end{array}$$

Simplify $\frac{1}{2}i^7$.

Simplify i^{42}.

-1

$42 \div 4 = 10$ R: 2

5-9 Operations with Complex Numbers

Recall that expressions in simplest form cannot have square roots in the denominator (Lesson 1-3). Because the imaginary unit represents a square root, you must rationalize any denominator that contains an imaginary unit. To do this, multiply the numerator and denominator by the complex conjugate of the denominator.

Helpful Hint

The complex conjugate of a complex number $a + bi$ is $a - bi$. (Lesson 5-5)

Like rationalizing a denominator, multiply both the denominator and numerator by the conjugate of the denominator to remove i.

Simplify.

$$\frac{(3-i)(2+i)}{(2-i)(2+i)}$$

$$\frac{6+3i-2i-i^2}{4+2i-2i-i^2}$$

$$\frac{6+i-(-1)}{4-(-1)}$$

$$\frac{7+i}{5}$$

Simplify.

$$\frac{(2+8i)}{(4-2i)}\frac{(4+2i)}{(4+2i)}$$

$$\frac{8+4i+32i+16i^2}{16+8i-8i-4i^2}$$

$$\frac{8+36i+(16)(-1)}{16-4(-1)}$$

$$\frac{-8+36i}{20}$$

Simplify.

$$\frac{3+8i}{-i}$$

Simplify.

$$\frac{3+10i}{5i}$$

Homework:

p. 377 #12-19, 24, 29, 36, 41, 45-48
p. 386 #36-51, 55-74, 109, 113-116