Warm Up

For each translation of the point (–2, 5), give the coordinates of the translated point.

1. 6 units down (-2,-1)

2. 3 units right (1,5)

For each function, evaluate *f*(–2), *f*(0), and *f*(3).

3. $f(x) = x^2 + 2x + 6$ $3^2+2(3)+6$

4. $f(x) = 2x^2 - 5x + 1$ $f(3)=21$

5-1 Using Transformations to Graph Quadratic Functions

Objectives

Transform quadratic functions.

Describe the effects of changes in the coefficients of $y = a(x - h)^2 + k$.

5-1 Using Transformations to Graph Quadratic Functions

Vocabulary

quadratic function
parabola
vertex of a parabola
vertex form

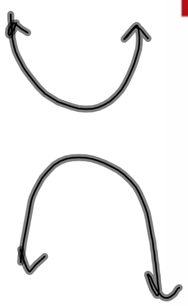
Holt Algebra 2

5-1 Using Transformations to Graph Quadratic Functions

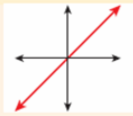
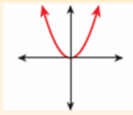
In Chapters 2 and 3, you studied linear functions of the form $f(x) = mx + b$. A **quadratic function** is a function that can be written in the form of $f(x) = a(x - h)^2 + k$ ($a \neq 0$). In a quadratic function, the variable is always squared. The table shows the linear and quadratic parent functions.

Holt Algebra 2

5-1 Using Transformations to Graph Quadratic Functions



Linear and Quadratic Parent Functions

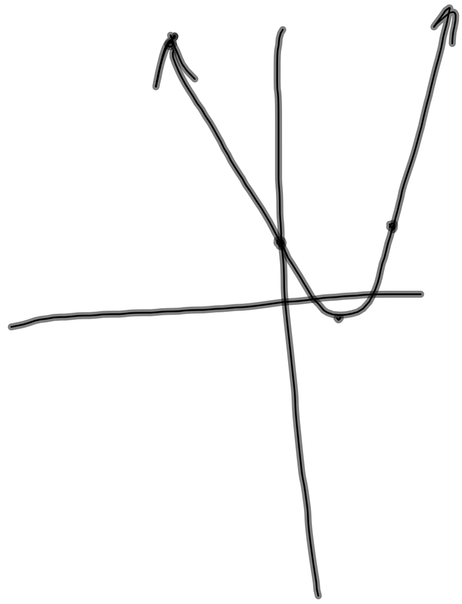
ALGEBRA	NUMBERS	GRAPH
Linear Parent Function $f(x) = x$		
	x: -2, -1, 0, 1, 2; $f(x) = x$: -2, -1, 0, 1, 2	
Quadratic Parent Function $f(x) = x^2$		
	x: -2, -1, 0, 1, 2; $f(x) = x^2$: 4, 1, 0, 1, 4	

Notice that the graph of the parent function $f(x) = x^2$ is a U-shaped curve called a **parabola**. As with other functions, you can graph a quadratic function by plotting points with coordinates that make the equation true.

Like all other graphs, start by making a table of values and plotting them to get your U shaped curve. Be sure to use both positive and negative x-values.

Graph $f(x) = x^2 - 4x + 3$ by using a table.

x	y	
-2	15	4+8+3
0	3	0-0+3
2	-1	4-8+3
4	3	16-16+3

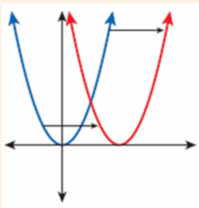
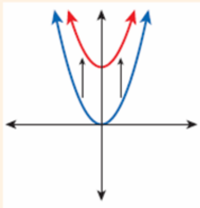


Graph $g(x) = -x^2 + 6x - 8$ by using a table.

5-1 Using Transformations to Graph Quadratic Functions

You can also graph quadratic functions by applying transformations to the parent function $f(x) = x^2$. Transforming quadratic functions is similar to transforming linear functions (Lesson 2-6).

Translations of Quadratic Functions

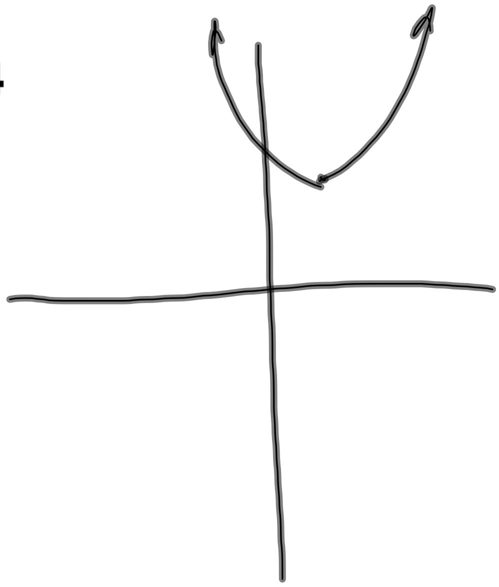
Horizontal Translations	Vertical Translations
Horizontal Shift of $\lvert h \rvert$ Units	Vertical Shift of $\lvert k \rvert$ Units
 $f(x) = x^2$ $f(x - h) = (x - h)^2$ Moves left for $h < 0$ Moves right for $h > 0$	 $f(x) = x^2$ $f(x) + k = x^2 + k$ Moves down for $k < 0$ Moves up for $k > 0$

Transformations do not go away. When you add/subtract to your x-value then you are going left/right. When you add/subtract from your function the you are going up/down.

Use the graph of $f(x) = x^2$ as a guide, describe the transformations and then graph each function.

$g(x) = (x - 2)^2 + 4$

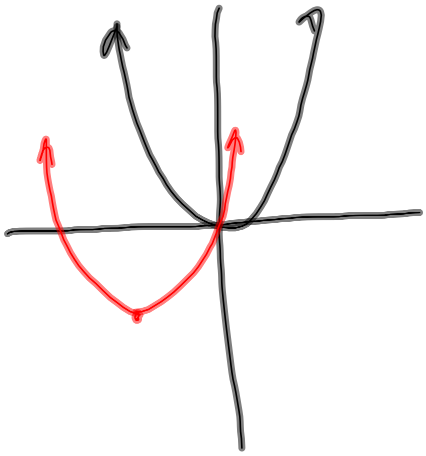
Right 2
Up 4



Use the graph of $f(x) = x^2$ as a guide, describe the transformations and then graph each function.

$g(x) = (x + 2)^2 - 3$

Left 2
Down 3



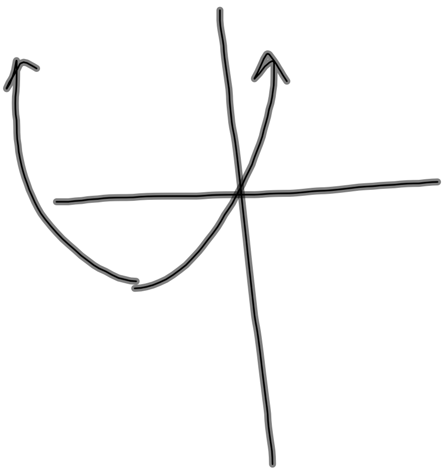
Using the graph of $f(x) = x^2$ as a guide, describe the transformations and then graph each function.

$g(x) = x^2 - 5$

Use the graph of $f(x) = x^2$ as a guide, describe the transformations and then graph each function.

$g(x) = (x + 3)^2 - 2$

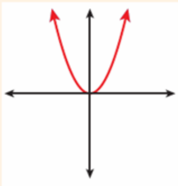
Left 3
Down 2



5-1 Using Transformations to Graph Quadratic Functions

Reflections

Reflection Across y-axis



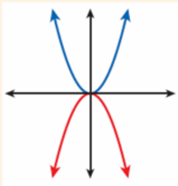
Input values change.

$f(x) = x^2$

$f(-x) = (-x)^2 = x^2$

The function $f(x) = x^2$ is its own reflection across the y-axis.

Reflection Across x-axis



Output values change.

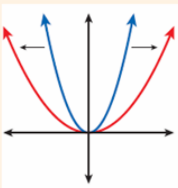
$f(x) = x^2$

$-f(x) = -(x^2)$
$= -x^2$

The function is flipped across the x-axis.

Stretches and Compressions

Horizontal Stretch/Compression by a Factor of $|b|$



Input values change.

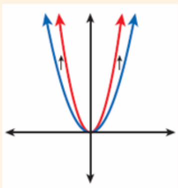
$f(x) = x^2$

$f\left(\frac{1}{b}x\right) = \left(\frac{1}{b}x\right)^2$

$|b| > 1$ stretches away from the y-axis.

$0 < |b| < 1$ compresses toward the y-axis.

Vertical Stretch/Compression by a Factor of $|a|$



Output values change.

$f(x) = x^2$

$a \cdot f(x) = ax^2$

$|a| > 1$ stretches away from the x-axis.

$0 < |a| < 1$ compresses toward the x-axis.

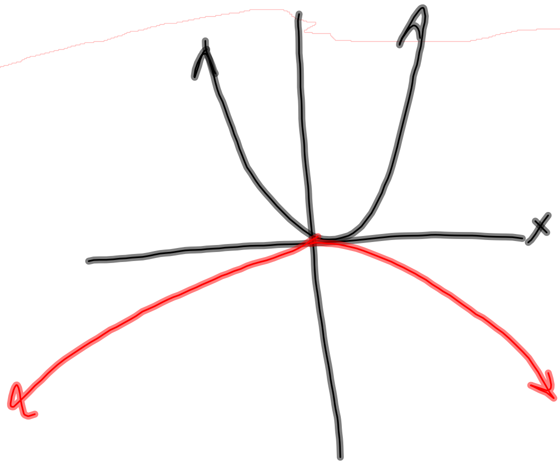
Identify each transformation to help sketch a graph more quickly.

Using the graph of $f(x) = x^2$ as a guide, describe the transformations and then graph each function.

$g(x) = -\frac{1}{4}x^2$

Reflect over x

Vertical Comp: $\frac{1}{4}$



Using the graph of $f(x) = x^2$ as a guide, describe the transformations and then graph each function.

$g(x) = (3x)^2$

Horz. Stretch: $\frac{1}{3}$

$\frac{1}{b} = 3$

$1 = 3b$

$b = \frac{1}{3}$

Using the graph of $f(x) = x^2$ as a guide, describe the transformations and then graph each function.

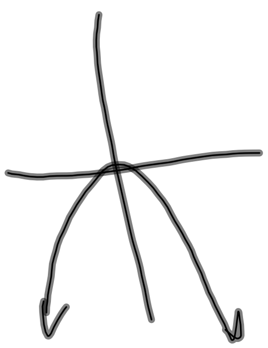
$g(x) = (2x)^2$

Using the graph of $f(x) = x^2$ as a guide, describe the transformations and then graph each function.

$g(x) = -\frac{1}{2}x^2$

Vert Stretch: $\frac{1}{2}$

Reflect over x



5-1 Using Transformations to Graph Quadratic Functions

If a parabola opens upward, it has a lowest point. If a parabola opens downward, it has a highest point. This lowest or highest point is the **vertex of the parabola**.

The parent function $f(x) = x^2$ has its vertex at the origin. You can identify the vertex of other quadratic functions by analyzing the function in *vertex form*. The **vertex form** of a quadratic function is $f(x) = a(x - h)^2 + k$, where a, h, and k are constants.

5-1 Using Transformations to Graph Quadratic Functions

Vertex Form of a Quadratic Function

$$f(x) = a(x - h)^2 + k$$

a indicates a reflection across the x-axis and/or a vertical stretch or compression.

h indicates a horizontal translation.

k indicates a vertical translation.

Because the vertex is translated h horizontal units and k vertical from the origin, the vertex of the parabola is at (h, k).

Helpful Hint

When the quadratic parent function $f(x) = x^2$ is written in vertex form, $y = a(x - h)^2 + k$, $a = 1$, $h = 0$, and $k = 0$.

Identify each value, plug them into the vertex formula and then simplify if you can.

Use the description to write the quadratic function in vertex form.

The parent function $f(x) = x^2$ is vertically stretched by a factor of $\frac{4}{3}$ and then translated 2 units left and 5 units down to create g.

$a: \frac{4}{3}$

$h: +2$

$k: -5$

$\frac{4}{3}(x+2)^2 - 5$

Use the description to write the quadratic function in vertex form.

The parent function $f(x) = x^2$ is vertically compressed by a factor of $\frac{1}{3}$ and then translated 2 units right and 4 units down to create *g*.

$a: \frac{1}{3}$

$h: -2$

$k: -4$

$\frac{1}{3}(x-2)^2-4$

Write a transformed parabola in vertex form and have a partner identify the transformations.

Use the description to write the quadratic function in vertex form.

The parent function $f(x) = x^2$ is reflected across the x-axis and translated 5 units left and 1 unit up to create g.

a: -1

h: +5

k: +1

$-(x+5)^2+1$

Homework:

p. 320 #17-30, 32, 39-41, 43, 46-49