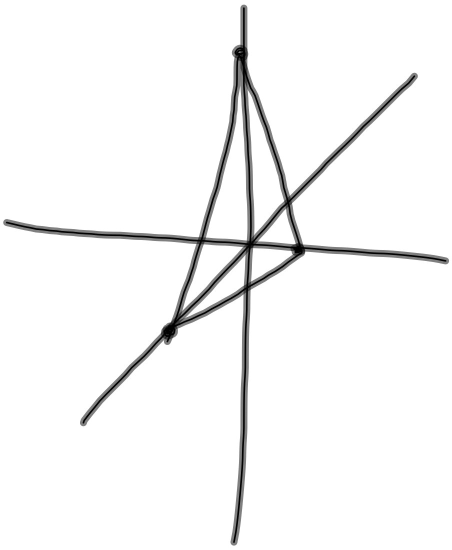
Warm up:
Graph the equation $3x + 2y + 6z = 12$

x-int: 4
y-int: 6
z-int: 2



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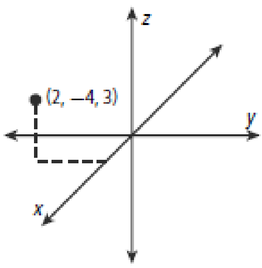

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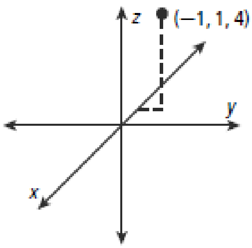

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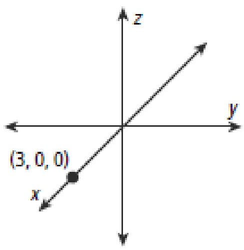

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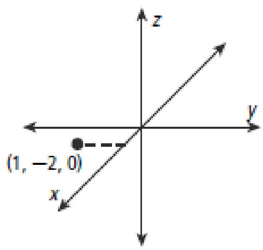

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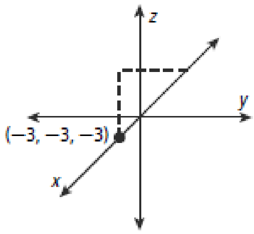

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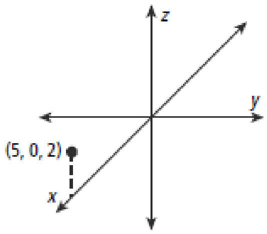

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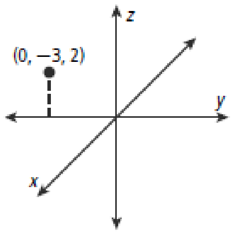

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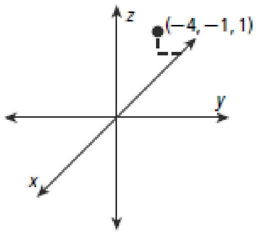
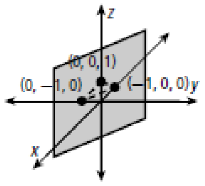
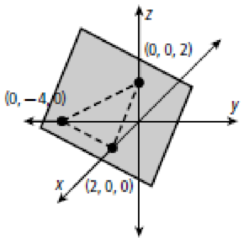


18. x-int.: $x + (0) - (0) = -1$
$x = -1$
y-int.: $(0) + y - (0) = -1$
$y = -1$
z-int.: $(0) + (0) - z = -1$
$z = 1$



19. x-int.: $2x - (0) + 2(0) = 4$
$2x = 4$
$x = 2$
y-int.: $2(0) - y + 2(0) = 4$
$-y = 4$
$y = -4$
z-int.: $2(0) - (0) + 2z = 4$
$2z = 4$
$z = 2$



20. x-int.: $2x + \frac{1}{2}(0) + (0) = -2$

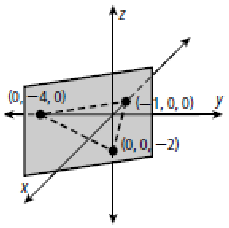
$$2x = -2$$
$$x = -1$$

y-int.: $2(0) + \frac{1}{2}y + (0) = -2$

$$\frac{1}{2}y = -2$$
$$y = -4$$

z-int.: $2(0) + \frac{1}{2}(0) + z = -2$

$$z = -2$$



21. x-int.: $5x + (0) - (0) = -5$

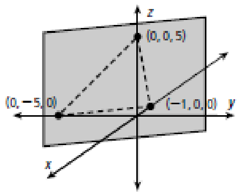
$$5x = -5$$
$$x = -1$$

y-int.: $5(0) + y - (0) = -5$

$$y = -5$$

z-int.: $5(0) + (0) - z = -5$

$$-z = -5$$
$$z = 5$$



22. x-int.: $8x + 6(0) + 4(0) = 24$

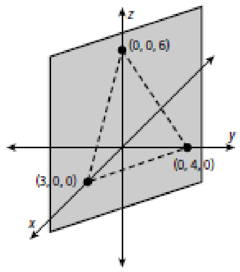
$$8x = 24$$
$$x = 3$$

y-int.: $8(0) + 6y + 4(0) = 24$

$$6y = 24$$
$$y = 4$$

z-int.: $8(0) + 6(0) + 4z = 24$

$$4z = 24$$
$$z = 6$$



23. x-int.: $3x - 3(0) + 2.5(0) = 7.5$

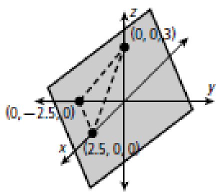
$$3x = 7.5$$
$$x = 2.5$$

y-int.: $3(0) - 3y + 2.5(0) = 7.5$

$$-3y = 7.5$$
$$y = -2.5$$

z-int.: $3(0) - 3(0) + 2.5z = 7.5$

$$2.5z = 7.5$$
$$z = 3$$



25. Let x be the number of free throws, y be the number of 2-pt. field goals, and z be the number of 3-pt. field goals.
$x + 2y + 3z = 60$
$20 + 2y + 3z = 60$
$2y + 3z = 40$
Possible answer:

Three-pointers	2	4	6	8	10
Two-pointers	17	14	11	8	5

32. Solution B is incorrect. To find the x-intercept, the z-value must equal 0.

33. A

34. C; intercepts are at (2, 0, 0), (0, 1, 0), and (0, 0, 3)

35. H

36. $\frac{3}{4}$; $5(0) - 2(0) - 4z = -3$
$-4z = -3$
$z = \frac{3}{4}$

3-6 Solving Linear Systems in Three Variables

Objectives

Represent solutions to systems of equations in three dimensions graphically.

Solve systems of equations in three dimensions algebraically.

3-6 Solving Linear Systems in Three Variables

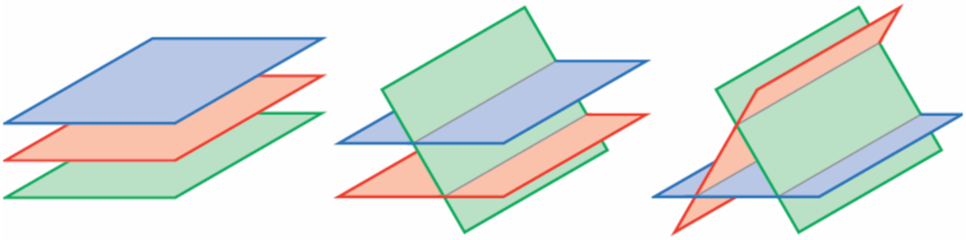
Systems of three equations with three variables are often called 3-by-3 systems. In general, to find a single solution to *any* system of equations, you need as many equations as you have variables.

3-6 Solving Linear Systems in Three Variables

Recall from Lesson 3-5 that the graph of a linear equation in three variables is a plane. When you graph a system of three linear equations in three dimensions, the result is three planes that may or may not intersect. The solution to the system is the set of points where all three planes intersect. These systems may have one, infinitely many, or no solution.

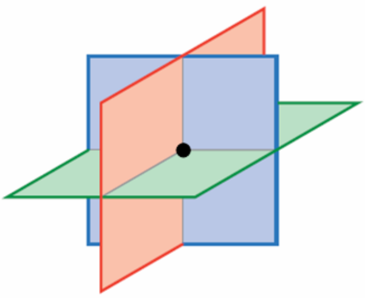
3-6 Solving Linear Systems in Three Variables

No Solutions
Inconsistent Systems

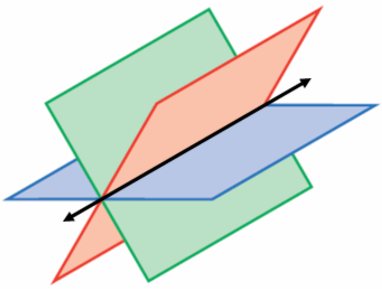


3-6 Solving Linear Systems in Three Variables

One Solution
Independent Systems



Infinitely Many Solutions
Dependent Systems



Holt Algebra 2
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3-6 Solving Linear Systems in Three Variables

Identifying the exact solution from a graph of a 3-by-3 system can be very difficult. However, you can use the methods of elimination and substitution to reduce a 3-by-3 system to a 2-by-2 system and then use the methods that you learned in Lesson 3-2.

Use elimination to solve the system of equations.

$$\begin{cases} 5x - 2y - 3z = -7 & (1) \\ 2x - 3y + z = -16 & (2) \\ 3x + 4y - 2z = 7 & (3) \end{cases}$$

(-3, 2, -4)

2(-3) - 3(2) + z = -16

-6 - 6 + z = -16

-12 + z = -16

z = -4

Use elimination to solve the system of equations.

$$\begin{cases} -x + y + 2z = 7 & ❶ \\ 2x + 3y + z = 1 & ❷ \\ -3x - 4y + z = 4 & ❸ \end{cases}$$

(-2, 1, 2)

3-6 Solving Linear Systems in Three Variables

You can also use substitution to solve a 3-by-3 system. Again, the first step is to reduce the 3-by-3 system to a 2-by-2 system.

The table shows the number of each type of ticket sold and the total sales amount for each night of the school play. Find the price of each type of ticket.

x y z

	Orchestra	Mezzanine	Balcony	Total Sales
Fri	200	30	40	$1470
Sat	250	60	50	$1950
Sun	150	30	0	$1050

Jada's chili won first place at the winter fair. The table shows the results of the voting.

How many points are first-, second-, and third-place votes worth?

Winter Fair Chili Cook-off				
Name	**1st Place**	**2nd Place**	**3rd Place**	**Total Points**
Jada	3	1	4	15
Maria	2	4	0	14
Al	2	2	3	13

3-6 Solving Linear Systems in Three Variables

The systems in Examples 1 and 2 have unique solutions. However, 3-by-3 systems may have no solution or an infinite number of solutions.

Remember!

Consistent means that the system of equations has at least one solution.

To see if a solution is consistent or not try to eliminate all of one variable and see how everything else relates.

Classify the system as consistent or inconsistent, and determine the number of solutions.

$$\begin{cases} 2x - 6y + 4z = 2 & ① \\ -3x + 9y - 6z = -3 & ② \\ 5x - 15y + 10z = 5 & ③ \end{cases}$$

Infinite soln

$6x - 18y + 12z = 6$
$-6x + 18y - 12z = -6$
$0 = 0$

$10x - 30y + 20z = 10$
$-10x + 30y - 20z = -10$
$0 = 0$

Classify the system, and determine the number of solutions.

$$\begin{cases} 3x - y + 2z = 4 & ① \\ 2x - y + 3z = 7 & ② \\ -9x + 3y - 6z = -12 & ③ \end{cases}$$

3-6 Solving Linear Systems in Three Variables

Check It Out! Example 3a Continued

Multiply equation ❷ by 3 and add to equation ❸.

❷ $3(2x - y + 3z = 7)$ → $6x - 3y + 9z = 21$

❸ $-9x + 3y - 6z = -12$ → $-9x + 3y - 6z = -12$

$$-3x + 3z = 9 \quad ❺$$

Now you have a 2-by-2 system.

$$\begin{cases} x - z = -3 & ❹ \\ -3x + 3z = 9 & ❺ \end{cases}$$

Classify the system, and determine the number of solutions.

$$\begin{cases} 2x - y + 3z = 6 & ❶ \\ 2x - 4y + 6z = 10 & ❷ \\ y - z = -2 & ❸ \end{cases}$$

Homework:

p. 224 #1-7, 16, 20-21

Extra Credit: 23(must show work)