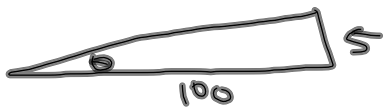
Warm Up:

A road has a 5% grade, which means that there is a 5 ft rise for 100 ft of horizontal distance. At what angle does the road rise from the horizontal? Round to the nearest tenth of a degree.



$\tan\theta = \frac{5}{100}$

$\tan^{-1}\left(\frac{5}{100}\right) = \theta = 2.8°$

13-5 The Law of Sines

Objectives

Determine the area of a triangle given side-angle-side information.

Use the Law of Sines to find the side lengths and angle measures of a triangle.

13-5 The Law of Sines

Capture

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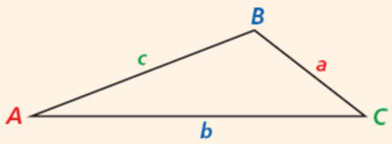
Area of a Triangle

For $\triangle ABC$,

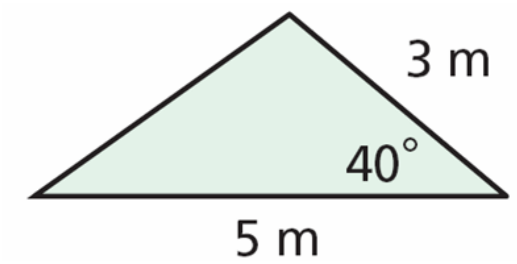
$$\text{Area} = \frac{1}{2}bc\sin A$$

$$\text{Area} = \frac{1}{2}ac\sin B$$

$$\text{Area} = \frac{1}{2}ab\sin C$$



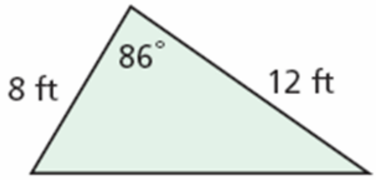
Find the area of the triangle. Round to the nearest tenth.



$\frac{1}{2} \cdot 3 \cdot 5 \cdot \sin 40$

$\approx 4.8\ m^2$

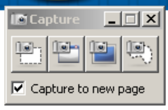
Find the area of the triangle. Round to the nearest tenth.



$\frac{1}{2} \cdot 8 \cdot 12 \cdot \sin 86$

$\approx 47.9 \text{ ft}^2$

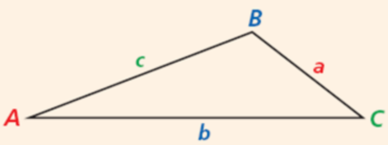
13-5 The Law of Sines



Law of Sines

For $\triangle ABC$, the Law of Sines states that

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$



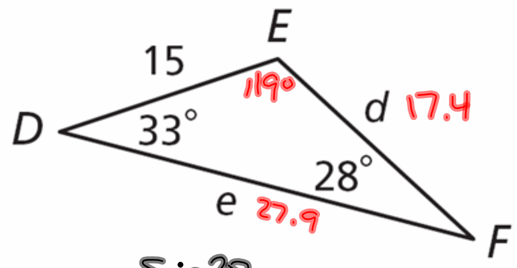
13-5 The Law of Sines

The Law of Sines allows you to solve a triangle as long as you know either of the following:

1. Two angle measures and any side length–angle-angle-side (AAS) or angle-side-angle (ASA) information
2. Two side lengths and the measure of an angle that is not between them–side-side-angle (SSA) information

Solve the triangle. Round to the nearest tenth.

$180-33-28$



$$\frac{\sin 28}{15}=\frac{\sin 33}{d}$$

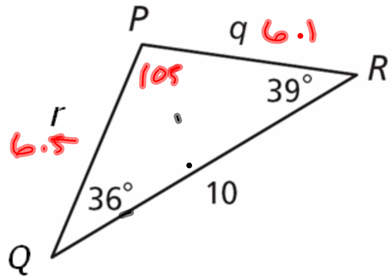
$$15\cdot\sin 33=d\cdot\sin 28$$

$$d=\frac{15\cdot\sin 33}{\sin 28}=17.4$$

$$\frac{\sin 28}{15}=\frac{\sin 119}{e}$$

$$e=\frac{15\cdot\sin 119}{\sin 28}=27.9$$

Solve the triangle. Round to the nearest tenth.



$180-39-36$
$=105°$

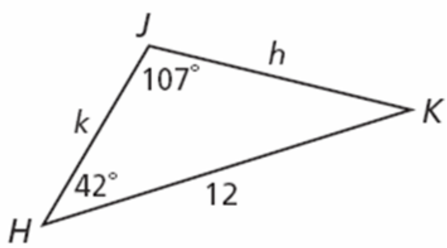
$\frac{\sin 39}{r} = \frac{\sin 105}{10}$

$r = 6.5$

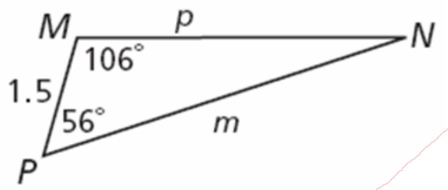
$\frac{\sin 36}{Q} = \frac{\sin 105}{10}$

$q = 6.1$

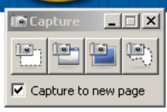
Solve the triangle. Round to the nearest tenth.



Solve the triangle. Round to the nearest tenth.



13-6 The Law of Cosines



Objectives

Use the Law of Cosines to find the side lengths and angle measures of a triangle.

Use Heron's Formula to find the area of a triangle.

13-6 The Law of Cosines

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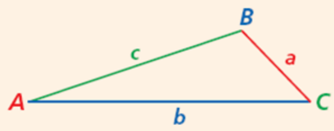
Law of Cosines

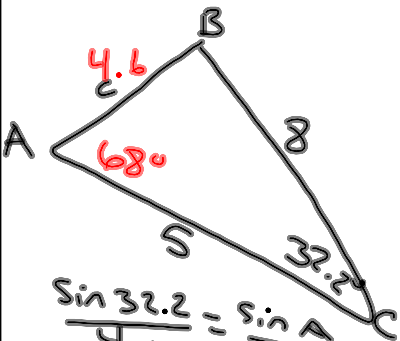
For $\triangle ABC$, the Law of Cosines states that

$$a^2 = b^2 + c^2 - 2bc\cos A.$$
$$b^2 = a^2 + c^2 - 2ac\cos B.$$
$$c^2 = a^2 + b^2 - 2ab\cos C.$$



Use the given measurements to solve $\triangle ABC$. Round to the nearest tenth.

$a = 8$, $b = 5$, $m\angle C = 32.2°$



$c^2 = 5^2 + 8^2 - 2(5)(8)\cos 32.2$

$c^2 = 89 - 67.69$

$c^2 = 21.3$

$c = 4.6$

$\frac{\sin 32.2}{4.6} = \frac{\sin A}{8}$

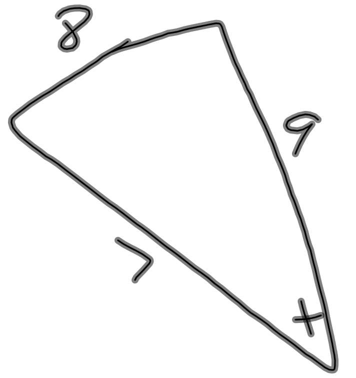
$8 \cdot \sin 32.2 = 4.6 \sin A$

$\sin^{-1}(\sin A) = \sin^{-1}\left(\frac{8 \cdot \sin 32.2}{4.6}\right)$

$A = 68°$

Use the given measurements to solve ΔABC. Round to the nearest tenth.

$a = 8, b = 9, c = 7$



$8^2 = 9^2 + 7^2 - 2(9)(7)\cos A$

$64 = 130 - 126\cos A$

$-66 = -126\cos A$

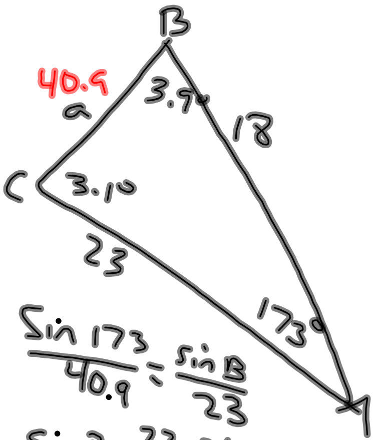
$\frac{-66}{-126} = \cos A$

$A = \cos^{-1}\left(\frac{-66}{-126}\right)$

$A = 58.4°$

Use the given measurements to solve ΔABC. Round to the nearest tenth.

$b = 23, c = 18, m\angle A = 173°$



$a^2 = 18^2 + 23^2 - 2(18)(23)\cos 173$

$a^2 = 1674$

$a = 40.9$

$\frac{\sin 173}{40.9} = \frac{\sin B}{23}$

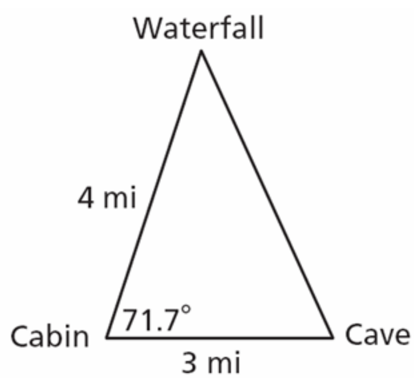
$\sin B = \frac{23 \cdot \sin 173}{40.9}$

$B = 3.9°$

Use the given measurements to solve $\triangle ABC$. Round to the nearest tenth.

$a = 35, b = 42, c = 50.3$

If a hiker travels at an average speed of 2.5 mi/h, how long will it take him to travel from the cave to the waterfall? Round to the nearest tenth of an hour.



$a^2 = 4^2 + 3^2 - 2(4)(3)\cos 71.7$

$a^2 = 25 - 24\cos 71.7$

$a^2 = 17$

$a = 4.2\text{mi}$

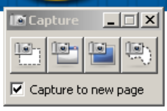
$\frac{4.2}{2.5} = 1.68\text{ hrs} = 1\text{ hr }40\text{min}$

A pilot is flying from Houston to Oklahoma City. To avoid a thunderstorm, the pilot flies 28° off the direct route for a distance of 175 miles. He then makes a turn and flies straight on to Oklahoma City. To the nearest mile, how much farther than the direct route was the route taken by the pilot?

34 mi furthe

$a^2 = 396^2 + 175^2 - 2(396)(175)\cos 28$

13-6 The Law of Cosines

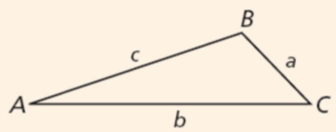


The Law of Cosines can be used to derive a formula for the area of a triangle based on its side lengths. This formula is called Heron's Formula.

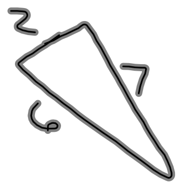
Heron's Formula

For $\triangle ABC$, where s is half of the perimeter of the triangle, or $\frac{1}{2}(a + b + c)$,

Area $= \sqrt{s(s-a)(s-b)(s-c)}$



A garden has a triangular flower bed with sides measuring 2 yd, 6 yd, and 7 yd. What is the area of the flower bed to the nearest tenth of a square yard?



$S = \frac{1}{2}(6+7+2)$

$S = 7.5$

$A = \sqrt{(7.5)(7.5-2)(7.5-6)(7.5-7)}$

$= \sqrt{(7.5)(5.5)(1.5)(.5)}$

$\approx 5.6 \text{ yd}^2$

The surface of a hotel swimming pool is shaped like a triangle with sides measuring 50 m, 28 m, and 30 m. What is the area of the pool's surface to the nearest square meter?

$S = \frac{1}{2}(50+30+28)$

$S = 54$

$A = \sqrt{54(4)(24)(28)}$

$= 381 \text{ m}^2$

Homework:

p. 963 #14-19, 25-29, 43-44, 46

p. 971 #9-14, 17-22, 33, 38-41