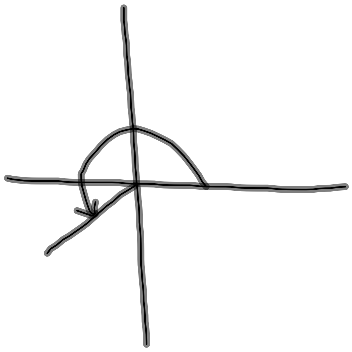
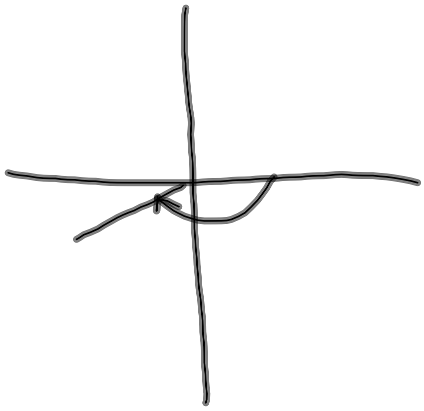
Warm Up:

Draw and angle in standard position with the given measure.

1. 210°



2. –160°



13-3 The Unit Circle

Objectives

Convert angle measures between degrees and radians.

Find the values of trigonometric functions on the unit circle.

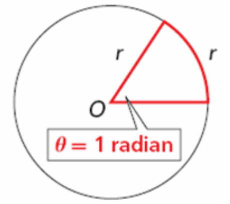
13-3 The Unit Circle

Vocabulary

radian
unit circle

13-3 The Unit Circle

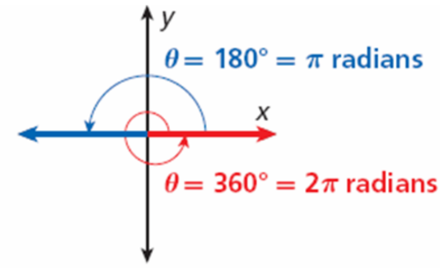
So far, you have measured angles in degrees. You can also measure angles in *radians*.



A **radian** is a unit of angle measure based on arc length. Recall from geometry that an *arc* is an unbroken part of a circle. If a central angle θ in a circle of radius r, then the measure of θ is defined as 1 radian.

13-3 The Unit Circle

The circumference of a circle of radius r is $2\pi r$. Therefore, an angle representing one complete clockwise rotation measures 2π radians. You can use the fact that 2π radians is equivalent to 360° to convert between radians and degrees.



13-3 The Unit Circle

Converting Angle Measures

DEGREES TO RADIANS	RADIANS TO DEGREES
Multiply the number of degrees by $\left(\frac{\pi \text{ radians}}{180°}\right)$.	Multiply the number of radians by $\left(\frac{180°}{\pi \text{ radians}}\right)$.

Convert each measure from degrees to radians or from radians to degrees.

A. – 60°

$$-\cancel{60}^{1}\left(\frac{\pi}{\cancel{180}_{3}}\right) = \frac{-\pi}{3}$$

B. $\frac{2\pi}{3}$ **radians**

$$\frac{2\cancel{\pi}}{\cancel{3}}\left(\frac{\cancel{180}^{60}}{\cancel{\pi}}\right) = 120^\circ$$

Convert each measure from degrees to radians or from radians to degrees.

a. 80°

$$\cancel{80}^{4}\left(\frac{\pi}{\cancel{180}_{9}}\right) = \frac{4\pi}{9}$$

b. $\frac{2\pi}{9}$ **radians**

$$\frac{2\cancel{\pi}}{\cancel{9}_{1}}\left(\frac{\cancel{180}^{20}}{\cancel{\pi}}\right) = 40^\circ$$

Convert each measure from degrees to radians or from radians to degrees.

c. –36°

$-36\left(\frac{\pi}{180}\right) = -\frac{\pi}{5}$

d. 4π radians

$4\cancel{\pi}\left(\frac{180}{\cancel{\pi}}\right) = 720°$

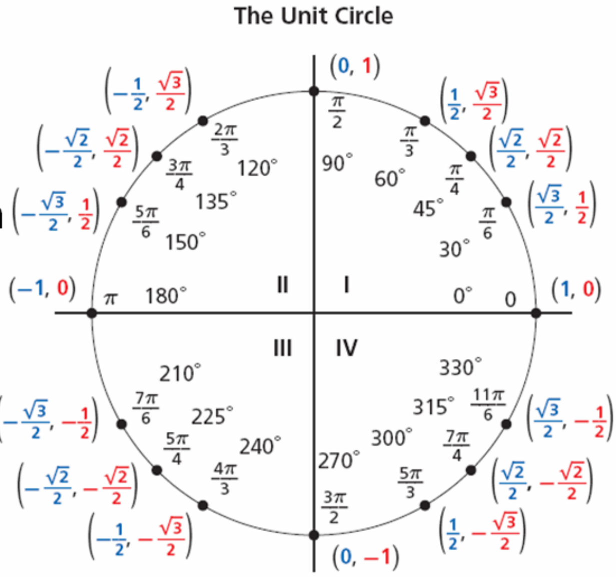
13-3 The Unit Circle

A **unit circle** is a circle with a radius of 1 unit. For every point $P(x, y)$ on the unit circle, the value of r is 1. Therefore, for an angle θ in the standard position:

$$\sin\theta = \frac{y}{r} = \frac{y}{1} = y$$

$$\cos\theta = \frac{x}{r} = \frac{x}{1} = x$$

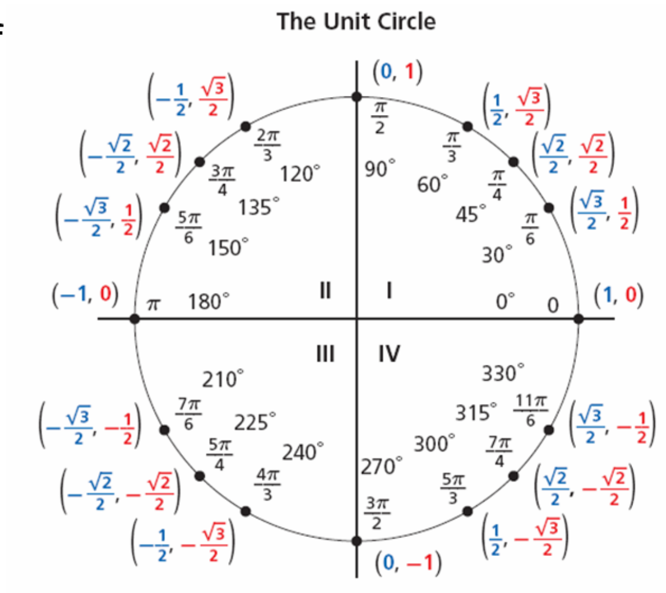
$$\tan\theta = \frac{y}{x}$$



13-3 The Unit Circle

So the coordinates of *P* can be written as (cosθ, sinθ).

The diagram shows the equivalent degree and radian measure of special angles, as well as the corresponding *x*- and *y*-coordinates of points on the unit circle.



Use the unit circle to find the exact value of each trigonometric function.

cos 225° $= -\frac{\sqrt{2}}{2}$

Use the unit circle to find the exact value of each trigonometric function.

$\tan \frac{5\pi}{6}$

$$\frac{\sin \frac{5\pi}{6}}{\cos \frac{5\pi}{6}} = \frac{\frac{1}{2} \cdot \frac{2}{\sqrt{3}}}{\frac{-\sqrt{3}}{2} \cdot \frac{2}{\sqrt{3}}} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

Use the unit circle to find the exact value of each trigonometric function.

$\sin 315°$

$$\frac{-\sqrt{2}}{2}$$

Use the unit circle to find the exact value of each trigonometric function.

tan 180°

$\frac{0}{-1} = 0$

Use the unit circle to find the exact value of each trigonometric function.

$\cos \frac{4\pi}{3}$

$-\frac{1}{2}$

13-3 The Unit Circle

You can use reference angles and Quadrant I of the unit circle to determine the values of trigonometric functions.

Trigonometric Functions and Reference Angles

To find the sine, cosine, or tangent of θ:	
Step 1	Determine the measure of the reference angle of θ.
Step 2	Use Quadrant I of the unit circle to find the sine, cosine, or tangent of the reference angle.
Step 3	Determine the quadrant of the terminal side of θ in standard position. Adjust the sign of the sine, cosine, or tangent based upon the quadrant of the terminal side.

13-3 The Unit Circle

The diagram shows how the signs of the trigonometric functions depend on the quadrant containing the terminal side of θ in standard position.

QII	$\sin\theta: +$ $\cos\theta: -$ $\tan\theta: -$	$\sin\theta: +$ $\cos\theta: +$ $\tan\theta: +$	QI
QIII	$\sin\theta: -$ $\cos\theta: -$ $\tan\theta: +$	$\sin\theta: -$ $\cos\theta: +$ $\tan\theta: -$	QIV

Use a reference angle to find the exact value of the sine, cosine, and tangent of 330°.

Sine: $-\frac{1}{2}$

Cos: $\frac{\sqrt{3}}{2}$

tan: $\frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}} = -\frac{1}{\sqrt{3}}$

Use a reference angle to find the exact value of the sine, cosine, and tangent of 270°.

Sin: -1

Cos: 0

tan: $\frac{-1}{0}$ = undefined

Use a reference angle to find the exact value of the sine, cosine, and tangent of each angle.

$\frac{11\pi}{6}$

Sin: $\frac{-1}{2}$

cos: $\frac{\sqrt{3}}{2}$

tan: $\frac{\frac{-1}{2}}{\frac{\sqrt{3}}{2}} = \frac{-1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{-1}{\sqrt{3}}$

Use a reference angle to find the exact value of the sine, cosine, and tangent of each angle.

–30°

13-3 The Unit Circle

If you know the measure of a central angle of a circle, you can determine the length s of the arc intercepted by the angle.

$$\frac{\text{radian measure of } \theta}{\text{radian measure of circle}} \rightarrow \frac{\theta}{2\pi} = \frac{s}{2\pi r} \leftarrow \frac{\text{arc length intercepted by } \theta}{\text{arc length intercepted by circle}}$$

$$\theta = \frac{s}{r}$$

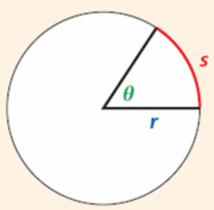
$$s = r\theta$$

13-3 The Unit Circle

Arc Length Formula

For a circle of radius r, the arc length s intercepted by a central angle θ (measured in radians) is given by the following formula.

$$s = r\theta$$



Holt Algebra 2
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A tire of a car makes 653 complete rotations in 1 min. The diameter of the tire is 0.65 m. To the nearest meter, how far does the car travel in 1 s?

653 rotations in 1min

so...

10.883 rotations in 1 sec

1 rotation = 2π

$\theta = 10.883 \cdot 2\pi = 21.77\pi$

$d = .65m$

$r = .325m$

$S = r\theta$

$S = (.325m)(21.77\pi)$

$S = 22.2\ m$

An minute hand on Big Ben's Clock Tower in London is 14 ft long. To the nearest tenth of a foot, how far does the tip of the minute hand travel in 1 minute?

$r = 14ft$

$\left(\frac{1}{60}\right)2\pi = \theta$

$\theta = \frac{1\pi}{30}$

$S = r\theta$

$S = (14)\left(\frac{\pi}{30}\right)$

$S \approx 1.5ft$

Homework:

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