Warm Up

(h, k) $(x-h)^2+(y-k)^2=r^2$

1. Write an equation for the circle with center (1, –5) and a radius of $\sqrt{10}$.

$(x-1)^2+(y+5)^2=10$

2. Write an equation for the circle with center (–4, 4) and containing the point (–1, 16).

$\sqrt{(-4+1)^2+(4-16)^2}$

$\sqrt{-3^2+-12^2}$

$\sqrt{9+144}$

$\sqrt{153}$

$(x+4)^2+(y-4)^2=153$

10-3 Ellipses

Objectives

Write the standard equation for an ellipse.

Graph an ellipse, and identify its center, vertices, co-vertices, and foci.

10-3 Ellipses

Vocabulary

ellipse
focus of an ellipse
major axis
vertices of an ellipse
minor axis
co-vertices of an ellipse

10-3 Ellipses

If you pulled the center of a circle apart into two points, it would stretch the circle into an ellipse.

An **ellipse** is the set of points $P(x, y)$ in a plane such that the sum of the distances from any point P on the ellipse to two fixed points F_1 and F_2, called the **foci** (singular: focus), is the constant sum $d = PF_1 + PF_2$. This distance d can be represented by the length of a piece of string connecting two pushpins located at the foci.

You can use the distance formula to find the constant sum of an ellipse.

Use the fact that an ellipse has a constant sum and therefore d=PF_1 + PF_2, where each part is using the distance formula with the point and the focus

Find the constant sum for an ellipse with foci F_1 (3, 0) and F_2 (24, 0) and the point on the ellipse (9, 8).

$d = PF_1 + PF_2$

$d = \sqrt{(9-3)^2 + (8-0)^2} + \sqrt{(9-24)^2 + (8-0)^2}$

$= \sqrt{6^2 + 8^2} + \sqrt{-15^2 + 8^2}$

$= \sqrt{36+64} + \sqrt{225+64}$

$= \sqrt{100} + \sqrt{289}$

$= 10 + 17$

$= 27$

Find the constant sum for an ellipse with foci F_1 (0, –8) and F_2 (0, 8) and the point on the ellipse (0, 10).

$d = PF_1 + PF_2$

$d = \sqrt{(0-0)^2+(10-{-8})^2} + \sqrt{(0-0)^2+(10-8)^2}$

$= \sqrt{18^2} + \sqrt{2^2}$

$= 18+2$

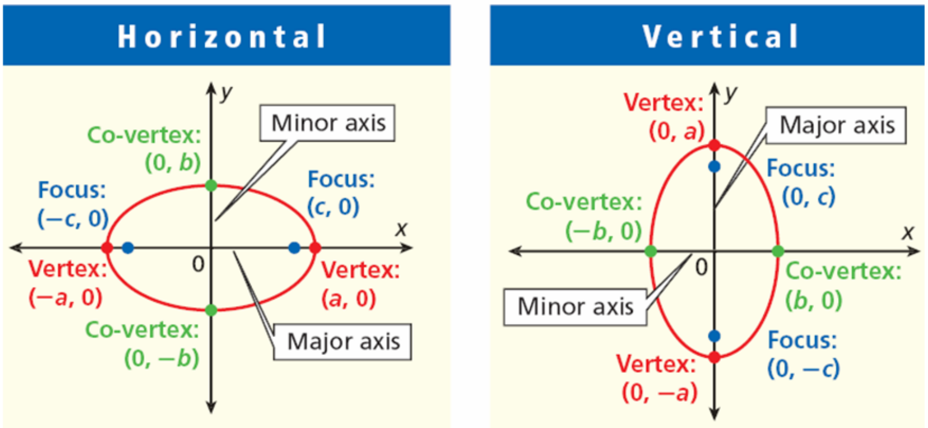
$= 20$

10-3 Ellipses

Instead of a single radius, an ellipse has two axes. The longer the axis of an ellipse is the **major axis** and passes through both foci. The endpoints of the major axis are the **vertices of the ellipse**. The shorter axis of an ellipse is the **minor axis**. The endpoints of the minor axis are the **co-vertices of the ellipse**. The major axis and minor axis are perpendicular and intersect at the center of the ellipse.

10-3 Ellipses

The standard form of an ellipse centered at (0, 0) depends on whether the major axis is horizontal or vertical.



10-3 Ellipses

The values a, b, and c are related by the equation $c^2 = a^2 - b^2$. Also note that the length of the major axis is $2a$, the length of the minor axis is $2b$, and $a > b$.

Standard Form for the Equation of an Ellipse Center at (0, 0)

MAJOR AXIS	HORIZONTAL	VERTICAL
Equation	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$
Vertices	$(a, 0), (-a, 0)$	$(0, a), (0, -a)$
Foci	$(c, 0), (-c, 0)$	$(0, c), (0, -c)$
Co-vertices	$(0, b), (0, -b)$	$(b, 0), (-b, 0)$

To write the equation of an ellipse there are 3 steps:

1) Pick the right equation, this is based on whether the major axis is horizontal or vertical
    -Do the vertices occur on the x or y axis?
2) Identify a and b values
3) Write the equation

Write an equation in standard form for each ellipse with center (0, 0).

Vertex at (6, 0); co-vertex at (0, 4)

$a=6$ Horz. Ellipse

$b=4$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\frac{x^2}{36}+\frac{y^2}{16}=1$$

Write an equation in standard form for each ellipse with center (0, 0).

Co-vertex at (5, 0); focus at (0, 3)

$a: \sqrt{34}$

$b: 5$

$c: 3$

$c^2 = a^2 - b^2$

$3^2 = a^2 - 5^2$

$9 = a^2 - 25$

$34 = a^2$

$a = \sqrt{34}$

Vertices on y axis

Vertical ellipse

$$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$$

$$\frac{y^2}{34} + \frac{x^2}{25} = 1$$

Write an equation in standard form for each ellipse with center (0, 0).

Vertex at (9, 0); co-vertex at (0, 5)

Horz

$a = 9$

$b = 5$

$$\frac{x^2}{81} + \frac{y^2}{25} = 1$$

Write an equation in standard form for each ellipse with center (0, 0).

Co-vertex at (4, 0); focus at (0, 3)

$a = 5$
$b = 4$
$c = 3$

vertical

$c^2 = a^2 - b^2$
$9 = a^2 - 16$
$a^2 = 25$
$a = 5$

$$\frac{y^2}{25} + \frac{x^2}{16} = 1$$

10-3 Ellipses

Ellipses may also be translated so that the center is not the origin.

Standard Form for the Equation of an Ellipse Center at (h, k)

MAJOR AXIS	HORIZONTAL	VERTICAL
Equation	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$
Vertices	$(h+a, k), (h-a, k)$	$(h, k+a), (h, k-a)$
Foci	$(h+c, k), (h-c, k)$	$(h, k+c), (h, k-c)$
Co-vertices	$(h, k+b), (h, k-b)$	$(h+b, k), (h-b, k)$

To graph identify the transformations that would move the center then accurately plot the vertices and connect in an elliptical shape.

h=-4 k=3

Graph the ellipse $\frac{(x+4)^2}{49}+\frac{(y-3)^2}{16}=1.$

Center (-4,3)

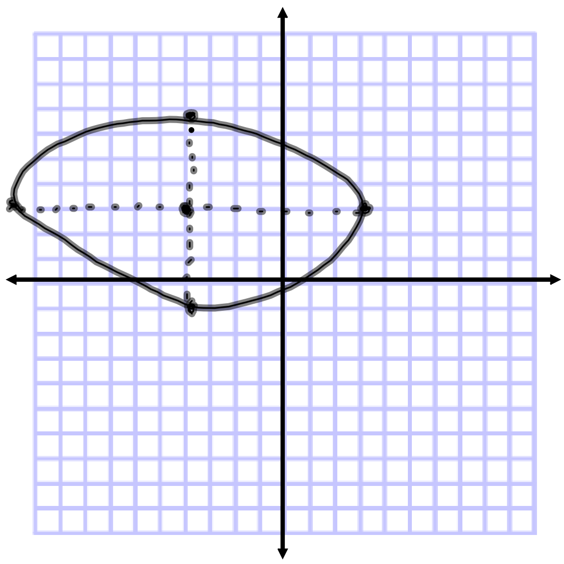
Horz.

$a^2=49$

$a=7$

$b^2=16$

$b=4$



Graph the ellipse $\frac{x^2}{64}+\frac{y^2}{25}=1.$

Center (0,0)

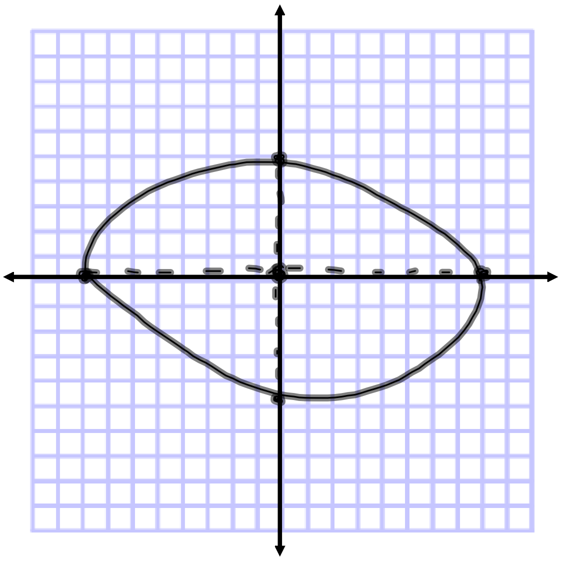
Horz.

$a^2=64$

$a=8$

$b^2=25$

$b=5$



Graph the ellipse.

$$\frac{(x-2)^2}{25}+\frac{(y-4)^2}{9}=1$$

Center (2,4)

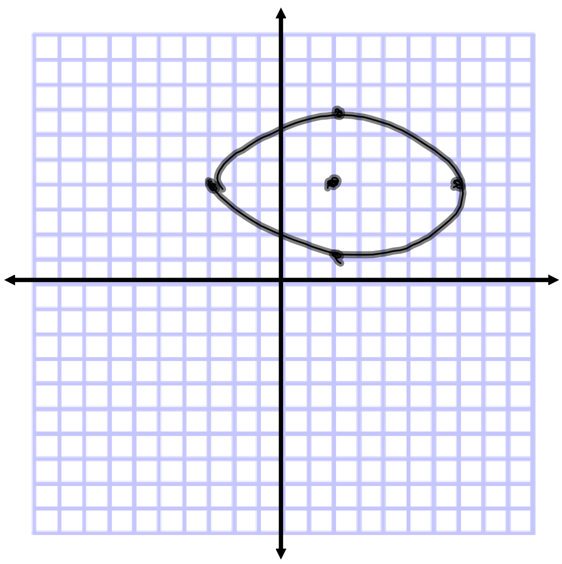
Horz.

$a^2=25$

$a=5$

$b^2=9$

$b=3$



A city park in the form of an ellipse with equation $\frac{x^2}{50} + \frac{y^2}{20} = 1$, measured in meters, is being renovated. The new park will have a length and width double that of the original park.

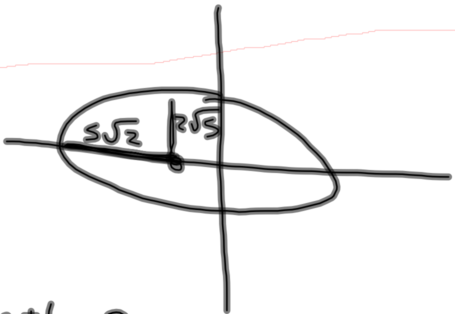
$a^2 = 50$

$a = \sqrt{50} = \sqrt{25} \cdot \sqrt{2}$

$5\sqrt{2}$

$b = \sqrt{20} = \sqrt{4} \cdot \sqrt{5}$

$2\sqrt{5}$



length of major $= 10\sqrt{2}$

length of minor $= 4\sqrt{5}$

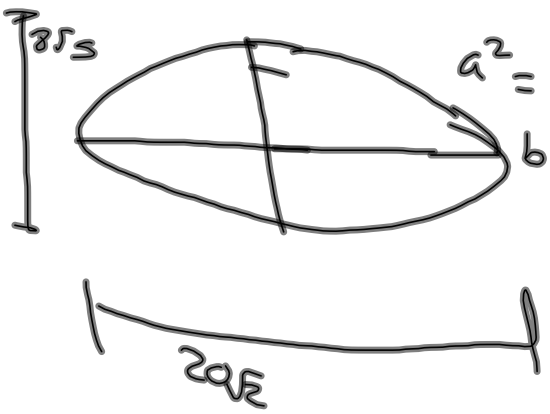
Find the dimensions of the new park.

major: $20\sqrt{2}$

minor: $8\sqrt{5}$

$a = 10\sqrt{2}$

$b = 4\sqrt{5}$



$a^2 = 100 \cdot 2 = 200$

$b^2 = 16 \cdot 5 = 80$

$\frac{x^2}{200} + \frac{y^2}{80} = 1$

B. Write an equation for the design of the new park.

Homework:

p. 740 #13-25, 27-29, 38-39, 42