Warm Up

Solve for *y*.

1. $x^2 + y^2 = 1$

$y^2 = -x^2 + 1$

$y = \pm\sqrt{-x^2+1}$

2. $4x^2 - 9y^2 = 1$

$-9y^2 = -4x^2 + 1$

$y^2 = \frac{-4x^2+1}{-9}$

$y = \pm\sqrt{\frac{-4x^2+1}{-9}}$

10-1 Introduction to Conic Sections

Objectives

Recognize conic sections as intersections of planes and cones.

Use the distance and midpoint formulas to solve problems.

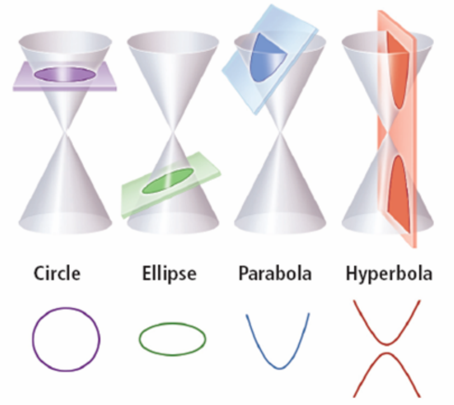
10-1 Introduction to Conic Sections

Vocabulary

conic section

10-1 Introduction to Conic Sections

In Chapter 5, you studied the parabola. The parabola is one of a family of curves called *conic sections*. **Conic sections** are formed by the intersection of a double right cone and a plane. There are four types of conic sections: circles, ellipses, hyperbolas, and parabolas.



Although the parabolas you studied in Chapter 5 are functions, most conic sections are not. This means that you often must use two functions to graph a conic section on a calculator.

10-1 Introduction to Conic Sections

A circle is defined by its center and its radius. An ellipse, an elongated shape similar to a circle, has two perpendicular axes of different lengths.

Remember!

When you take the square root of both sides of an equation, remember that you must include the positive and negative roots.

Holt Algebra 2

To graph equations on a calculator:

1) Solve for y, make sure to include the positive and negative roots
2) Use the y= function on the calculator to see the curve

Graph each equation on a graphing calculator. Identify each conic section. Then describe the center and intercepts.

$(x-1)^2+(y-1)^2=1$

Circle

$-(x-1)^2$ $-(x-1)^2$

Center (1,1)

$\sqrt{(y-1)^2}=\sqrt{-(x-1)^2}$

Intercept: (1,0) (0,1)

$y-1=\pm\sqrt{1-(x-1)^2}$

$y=\pm\sqrt{1-(x-1)^2}+1$

$4x^2+25y^2=100$

$25y^2=100-4x^2$

$y^2=\frac{100-4x^2}{25}$

Ellipse $y=\pm\sqrt{\frac{100-4x^2}{25}}$

Center: (0,0)

Intercepts: (0,2) (0,-2)

(5,0) (-5,0)

Graph each equation on a graphing calculator. Identify each conic section. Then describe the center and intercepts.

$x^2 + y^2 = 49$

$y = \pm\sqrt{49 - x^2}$

Circle

(0,7) (0,-7)
(7,0) (-7,0)

$9x^2 + 25y^2 = 225$

10-1 Introduction to Conic Sections

A parabola is a single curve, whereas a hyperbola has two congruent branches. The equation of a parabola usually contains either an x^2 term or a y^2 term, but not both. The equations of the other conics will usually contain both x^2 and y^2 terms.

Helpful Hint

Because hyperbolas contain two curves that open in opposite directions, classify them as opening horizontally, vertically, or neither.

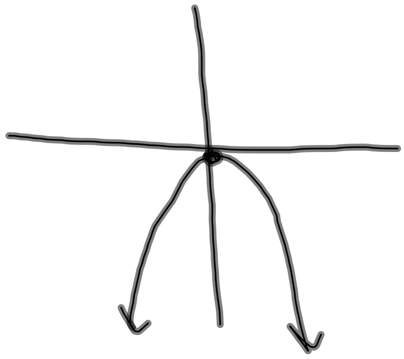
Graph each equation on a graphing calculator. Identify each conic section. Then describe the vertices and the direction that the graph opens.

$y = -\frac{1}{2}x^2$

Parabola

Opens down

(0,0)



$y^2 - x^2 = 9$

$y = \pm\sqrt{x^2 + 9}$

Hyperbola

(0,3) (0,-3)

Opens vertically

Graph each equation on a graphing calculator. Identify each conic section. Then describe the vertices and the direction that the graph opens.

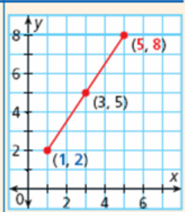
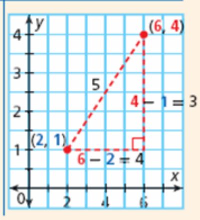
$2y^2 = x$

$$x^2 - y^2 = 16$$

10-1 Introduction to Conic Sections

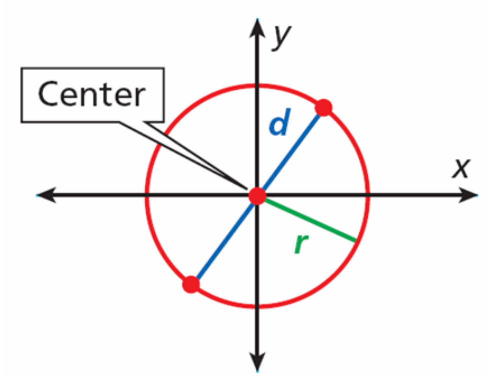
Every conic section can be defined in terms of distances. You can use the Midpoint and Distance Formulas to find the center and radius of a circle.

Midpoint and Distance Formulas

FORMULA	EXAMPLE	GRAPH
The **midpoint** (x_M, y_M) of the segment with endpoints (x_1, y_1) and (x_2, y_2) is $(x_M, y_M) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.	The midpoint of the segment with endpoints $(1, 2)$ and $(5, 8)$ is $\left(\frac{1+5}{2}, \frac{2+8}{2}\right) = (3, 5)$.	
The **distance** d between the points with coordinates (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.	The distance between the points $(2, 1)$ and $(6, 4)$ is $\sqrt{(6-2)^2 + (4-1)^2} = 5$.	

10-1 Introduction to Conic Sections

Because a diameter must pass through the center of a circle, the midpoint of a diameter is the center of the circle. The radius of a circle is the distance from the center to any point on the circle and equal to half the diameter.



Helpful Hint

The midpoint formula uses averages. You can think of x_M as the average of the x-values and y_M as the average of the y-values.

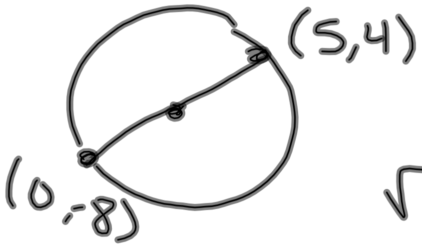
Holt Algebra 2

A radius is how much of a diameter?

$\frac{1}{2}$ of diameter

To find a length, use the distance formula!

Find the center and radius of a circle that has a diameter with endpoints (5, 4) and (0, –8).



Center:

$\left(\frac{5+0}{2}, \frac{-8+4}{2}\right)$

$\left(\frac{5}{2}, -2\right)$

Distance of r:

$\sqrt{(5-\frac{5}{2})^2+(4--2)^2}$

$\sqrt{(\frac{5}{2})^2+(6)^2}$

$\sqrt{\frac{25}{4}+36}$

$\sqrt{42.25}$

$=6.5$

Find the center and radius of a circle that has a diameter with endpoints (2, 6) and (14, 22).

Center:

$\left(\frac{2+14}{2}, \frac{6+22}{2}\right)$

$(8, 14)$

r:

$\frac{\sqrt{(14-2)^2+(22-6)^2}}{2}$

$\frac{\sqrt{12^2+16^2}}{2}$

$\frac{20}{2}=10$

Find the center and radius of a circle that has a diameter with endpoints (3, 7) and (−2, −5).

Center:

$\left(\frac{3+-2}{2}, \frac{7+-5}{2}\right)$

$\left(\frac{1}{2}, 1\right)$

$(3,7)\ (\frac{1}{2},1)$

$\sqrt{(3-\frac{1}{2})^2 + (7-1)^2}$

$\sqrt{(2.5)^2 + (6)^2}$

$\sqrt{6.25+36}$

$= 6.5$

Homework:

p. 726 #14-34(even), 37-40, 43-44, 46-48, 50-53